Null Hypothesis Significance Testing: A Review of an Old and Continuing Controversy

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Null hypothesis significance testing (NHST) is arguably the most widely used approach to hypothesis evaluation among behavioral and social scientists. It is also very controversial. A major concern expressed by critics is that such testing is misunderstood by many of those who use it. Several other objections to its use have also been raised. In this article the author reviews and comments on the claimed misunderstandings as well as on other criticisms of the approach, and he notes arguments that have been advanced in support of NHST. Alternatives and supplements to NHST are considered, as are several related recommendations regarding the interpretation of experimental data. The concluding opinion is that NHST is easily misunderstood and misused but that when applied with good judgment it can be an effective aid to the interpretation of experimental data.

Null hypothesis statistical testing (NHST) is arguably the most widely used method of analysis of data collected in psychological experiments and has been so for about 70 years. One might think that a method that had been embraced by an entire research community would be well understood and noncontroversial after many decades of constant use. However, NHST is very controversial. Criticism of the method, which essentially began with the introduction of the technique (Pearce, 1992), has waxed and waned over the years; it has been intense in the recent past. Apparently, controversy regarding the idea of NHST more generally extends back more than two and a half centuries (Hacking, 1965).

The purpose of this article is to review the controversy critically, especially the more recent contributions to it. The motivation for this exercise comes from the frustration I have felt as the editor of an empirical journal in dealing with submitted manu-

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1 Null hypothesis statistical significance testing is abbreviated in the literature as NHST (with the S sometimes representing statistical and sometimes significance), as NHSTP (P for procedure), NHTP (null hypothesis testing procedure), NHT (null hypothesis testing), ST (significance testing), and possibly in other ways. I use NHST here because I think it is the most widely used abbreviation.

2 One of the people who gave me very useful feedback on a draft of this article questioned the accuracy of my claim that NHST is very controversial. "I think the impression that NHST is very controversial comes from focusing on the collection of articles you review—the product of a batch of authors arguing with each other and rarely even glancing at actual researchers outside the circle except to lament how little the researchers seem to benefit from all the sage advice being aimed by the debaters at both sides of almost every issue." The implication seems to be that the "controversy" is largely a manufactured one, of interest primarily—if not only—to those relatively few authors who benefit from keeping it alive. I must admit that this comment, from a psychologist for whom I have the highest esteem, gave me some pause about the wisdom of investing more time and effort in this article. I am convinced, however, that the controversy is real enough and that it deserves more attention from users of NHST than it has received.
scripts, the vast majority of which report the use of NHST. In attempting to develop a policy that would help ensure the journal did not publish egregious misuses of this method, I felt it necessary to explore the controversy more deeply than I otherwise would have been inclined to do. My intent here is to lay out what I found and the conclusions to which I was led.3

Some Preliminaries

Null hypothesis has been defined in a variety of ways. The first two of the following definitions are from mathematics dictionaries, the third from a dictionary of statistical terms, the fourth from a dictionary of psychological terms, the fifth from a statistics text, and the sixth from a frequently cited journal article on the subject of NHST:

A particular statistical hypothesis usually specifying the population from which a random sample is assumed to have been drawn, and which is to be nullified if the evidence from the random sample is unfavorable to the hypothesis, i.e., if the random sample has a low probability under the null hypothesis and a higher one under some admissible alternative hypothesis. (James & James, 1959, p. 195)

1. The residual hypothesis that cannot be rejected unless the test statistic used in the hypothesis testing problem lies in the critical region for a given significance level. 2. in particular, especially in psychology, the hypothesis that certain observed data are a merely random occurrence. (Borowski & Borwein, 1991, p. 411)

A particular hypothesis under test, as distinct from the alternative hypotheses which are under consideration. (Kendall & Buckland, 1957)

The logical contradictory of the hypothesis that one seeks to test. If the null hypothesis can be proved false, its contradictory is thereby proved true. (English & English, 1958, p. 350)

Symbolically, we shall use $H_0$ (standing for null hypothesis) for whatever hypothesis we shall want to test and $H_A$ for the alternative hypothesis. (Freund, 1962, p. 238) Except in cases of multistage or sequential tests, the acceptance of $H_0$ is equivalent to the rejection of $H_A$, and vice versa. (p. 250)

The null hypothesis states that the experimental group and the control group are not different with respect to [a specified property of interest] and that any difference found between their means is due to sampling fluctuation. (Carver, 1978, p. 381)

It is clear from these examples—and more could be given—that null hypothesis has several connotations.

For present purposes, one distinction is especially important. Sometimes null hypothesis has the relatively inclusive meaning of the hypothesis whose nullification, by statistical means, would be taken as evidence in support of a specified alternative hypothesis (e.g., the examples from English & English, 1958; Kendall & Buckland, 1957; and Freund, 1962, above). Often—perhaps most often—as used in psychological research, the term is intended to represent the hypothesis of "no difference" between two sets of data with respect to some parameter, usually their means, or of "no effect" of an experimental manipulation on the dependent variable of interest. The quote from Carver (1978) illustrates this meaning.

Given the former connotation, the null hypothesis may or may not be a hypothesis of no difference or of no effect (Bakan, 1966). The distinction between these connotations is sometimes made by referring to the second one as the nil null hypothesis or simply the nil hypothesis; usually the distinction is not made explicitly, and whether null is to be understood to mean nil null must be inferred from the context. The distinction is an important one, especially relative to the controversy regarding the merits or shortcomings of NHST inasmuch as criticisms that may be valid when applied to nil hypothesis testing are not necessarily valid when directed at null hypothesis testing in the more general sense.

Application of NHST to the difference between two means yields a value of $p$, the theoretical probability that if two samples of the size of those used had been drawn at random from the same population, the statistical test would have yielded a statistic (e.g., $t$) as large or larger than the one obtained. A specified significance level conventionally designated $\alpha$ (alpha) serves as a decision criterion, and the null hypothesis would be rejected.

3Since this article was submitted to Psychological Methods for consideration for publication, the American Psychological Association's Task Force on Statistical Inference (TFSI) published a report in the American Psychologist (Wilkinson & TFSI, 1999) recommending guidelines for the use of statistics in psychological research. This article was written independently of the task force and for a different purpose. Having now read the TFSI report, I like to think that the present article reviews much of the controversy that motivated the convening of the TFSI and the preparation of its report. I find the recommendations in that report very helpful, and I especially like the admonition not to rely too much on statistics in interpreting the results of experiments and to let statistical methods guide and discipline thinking but not determine it.
is rejected only if the value of \( p \) yielded by the test is not greater than the value of \( \alpha \). If \( \alpha \) is set at .05, say, and a significance test yields a value of \( p \) equal to or less than .05, the null hypothesis is rejected and the result is said to be statistically significant at that level.

According to most textbooks, the logic of NHST admits of only two possible decision outcomes: rejection (at a specified significance level) of the hypothesis of no difference, and failure to reject this hypothesis (at that level). Given the latter outcome, one is justified in saying only that a significant difference was not found; one does not have a basis for concluding that the null hypothesis is true (that the samples were drawn from the same population with respect to the variable of interest). Inasmuch as the null hypothesis may be either true or false and it may either be rejected or fail to be rejected, any given instance of NHST admits of four possible outcomes, as shown in Table 1.

There are two ways to be right: rejecting the null hypothesis when it is false (when the samples were drawn from different populations) and failing to reject it when it is true (when the samples were drawn from the same population). There are also two ways to be wrong: rejecting the null hypothesis when it is true and failing to reject it when it is false. The first of these two ways to be wrong is usually referred to as a Type I error, and the second as a Type II error, after Neyman and Pearson (1933a).

By definition, a Type I error can be made only when the null hypothesis is true. The value of \( p \) that is obtained as the result of NHST is the probability of a Type I error on the assumption that the null hypothesis is true. The unconditional probability of the occurrence of a Type I error is the product of \( p \) and the probability that the null hypothesis is true. Failure to make this distinction between the probability of a Type I error conditional on the null being true and the unconditional probability of a Type I error has been the basis of some confusion, which I discuss further below.

Similarly, by definition, a Type II error can be made only when the null hypothesis is false. The probability of occurrence of a Type II error—when the null hypothesis is false—is usually referred to as \( \beta \) (beta). The unconditional probability of occurrence of a Type II error is the product of \( \beta \) and the probability that the null hypothesis is false. \( \beta \) is generally assumed to be larger than \( p \) but not known precisely.

Closely related to the concept of statistical significance is that of power (Chase & Tucker, 1976; Cohen, 1977, 1988; Rossi, 1990), which is defined as \( 1 - \beta \). Power is the probability of rejecting the null hypothesis conditional on its being false, that is, the probability of detecting an effect given that there is one, or the probability of accepting the alternative hypothesis conditional on its being true. It is possible to compute power to detect an effect of a hypothesized size, and this is what is typically done: One determines the probability that a specified sample size would yield significance at a specified alpha level given an effect of a hypothesized magnitude.

The use of NHST in psychology has been guided by a greater tolerance for failing to reject the null hypothesis when it is false (Type II error) than for rejecting it when it is true (Type I error). This preference is reflected in the convention of selecting a decision criterion (confidence level) such that one will reject the hypothesis of no difference only if the observed difference would be theoretically unlikely—a probability of, say, less than .05 or less than .01—to be obtained by chance from samples drawn from the same population. A decision criterion of .05 is intended to represent a strong bias against the making of a Type I error, and a criterion of .01 is an even stronger one. (The assumption that the intention has been realized to the extent generally believed has been challenged; I return to this point below.) The approach of biasing against Type I error is intended to be conservative in the sense of beginning with an assumption of no difference and giving up that assumption only on receipt of strong evidence that it is false. This conservativeness can be seen as in keeping with the spirit of Occam's razor, according to which entities (theories, effects) should not be multiplied unnecessarily (Rindskopf, 1997).

The rationale for conservatism in statistical testing for sample differences is strikingly similar to the one that guides the proceedings in a U.S. court of law. The rule in a criminal trial is that the defendant is to be presumed innocent and can be judged guilty only if the prosecution proves guilt beyond a reasonable doubt. Furthermore, the trial can yield one of only two
possible verdicts: guilty or not guilty. Not guilty, in this context, is not synonymous with innocent; it means only that guilt was not demonstrated with a high degree of certainty. Proof of innocence is not a requirement for this verdict; innocence is a presumption and, like the null hypothesis, it is to be rejected only on the basis of compelling evidence that it is false. The asymmetry in this case reflects the fact that the possibility of letting a guilty party go free is strongly preferred to the possibility of convicting someone who is innocent. This analogy has been discussed by Feinberg (1971).

Statistical significance tests of differences between means are usually based on comparison of a measure of variability across samples with a measure of variability within samples, weighted by the number of items in the samples. To be statistically significant, a difference between sample means has to be large if the within-sample variability is large and the number of items in the samples is small; however, if the within-sample variability is small and the number of items per sample is large, even a very small difference between sample means may attain statistical significance. This makes intuitive sense. The larger the size of a sample, the more confidence one is likely to have that it faithfully reflects the characteristics of the population from which it was drawn. Also, the less the members of the same sample differ among each other with respect to the measure of interest, the more impressive the differences between samples will be.

Sometimes a distinction is made between rejection-support (RS) and acceptance-support (AS) NHST (Binder, 1963; Steiger & Fouladi, 1997). The distinction relates to Meehl's (1967, 1997) distinction between strong and weak uses of statistical significance tests in theory appraisal (more on that below). In RS-NHST the null hypothesis represents what the experimenter does not believe, and rejection of it is taken as support of the experimenter's theoretical position, which implies that the null is false. In AS-NHST the null hypothesis represents what the experimenter believes, and acceptance of it is taken as support for the experimenter's view. (A similar distinction, between a situation in which one seeks to assert that an effect in a population is large and a situation in which one seeks to assert that an effect in a population is small, has been made in the context of Bayesian data analysis [Rouanet, 1996].)

RS testing is by far the more common of the two types, and the foregoing comments, as well as most of what follows, apply to it. In AS testing, Type I and Type II errors have meanings opposite the meanings of these terms as they apply to RS testing. Examples of the use of AS in cognitive neuroscience are given by Bookstein (1998). AS testing also differs from RS in a variety of other ways that will not be pursued here.

The Controversial Nature of NHST

Although NHST has played a central role in psychological research—a role that was foreshadowed by Fisher's (1935) observation that every experiment exists to give the facts a chance of disproving the null hypothesis—it has been the subject of much criticism and controversy (Kirk, 1972; Morrison & Henkel, 1970). In a widely cited article, Rozeboom (1960) argued that despite the awesome pre-eminence this method has attained in our journals and textbooks of applied statistics, it is based upon a fundamental misunderstanding of the nature of rational inference, and is seldom if ever appropriate to the aims of scientific research. (p. 417)

The passage of nearly four decades has not tempered Rozeboom's disdain for NHST (Rozeboom, 1997). In another relatively early critique of NHST, Eysenck (1960) made a case for not using the term significance in reporting the results of research. C. A. Clark (1963) argued that statistical significance tests do not provide the information scientists need and that the null hypothesis is not a sound basis for statistical investigation.

Other behavioral and social scientists have criticized the practice, which has long been the convention within these sciences, of making NHST the primary method of research and often the major criterion for the publication of the results of such research (Bakan, 1966; Brewer, 1985; Cohen, 1994; Cronbach, 1975; Dracup, 1995; Falk, 1986; Falk & Greenbaum, 1995; Folger, 1989; Gigerenzer & Murray, 1987; Grant, 1962; Gutman, 1977, 1985; Jones, 1955; Kirk, 1996; Kish, 1959; Lunt & Livingstone, 1989; Lykkken, 1968; McNemar, 1960; Meehl, 1967, 1990a, 1990b; Oakes, 1986; Pedhazur & Schmelkin, 1991; Pollard, 1993; Rossi, 1990; Sodlemeier & Gigerenzer, 1989; Shaver, 1993; Shrout, 1997; Signorelli, 1974; Thompson, 1993, 1996, 1997). An article that stimulated numerous others was that of Cohen (1994). (See commentary in American Psychologist [Baril and Cannon, 1995; Frick, 1995b; Hubbard, 1995; McGraw, 1995;
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Parker, 1995; Svyantek and Ekeberg, 1995] and the response by Cohen, 1995.)

Criticism has often been severe. Bakan (1966), for example, referred to the use of NHST in psychological research as “an instance of a kind of essential mindlessness in the conduct of research” (p. 436). Carver (1978) said of NHST that it “has involved more fantasy than fact” and described the emphasis on it as representing “a corrupt form of the scientific method” (p. 378). Lakatos (1978) was led by the reading of Meehl (1967) and Lykken (1968) to wonder whether the function of statistical techniques in the social sciences is not primarily to provide a machinery for producing phony corroborations and thereby a semblance of “scientific progress” where, in fact, there is nothing but an increase in pseudo-intellectual garbage. (p. 88)

Gigerenzer (1998a) argued that the institutionalization of NHST has permitted surrogates for theories (one-word explanations, redescriptions, vague dichotomies, data fitting) to flourish in psychology:

Null hypothesis testing provides researchers with no incentive to specify either their own research hypotheses or competing hypotheses. The ritual is to test one’s unspecified hypothesis against “chance,” that is, against the null hypothesis that postulates “no difference between the means of two populations” or “zero correlation.” (p. 200)

Rozeboom (1997) has referred to NHST as “surely the most bone-headedly misguided procedure ever institutionalized in the rote training of science students” (p. 335).

Excepting the last two, these criticisms predate the ready availability of software packages for doing statistical analyses; some critics believe the increasing prevalence of such software has exacerbated the problem. Estes (1997a) has pointed out that statistical results are meaningful only to the extent that both author and reader understand the basis of their computation, which often can be done in more ways than one; mutual understanding can be impeded if either author or reader is unaware of how a program has computed a statistic of a given name. Thompson (1998) claimed that “most researchers mindlessly test only nulls of no difference or of no relationship because most statistical packages only test such hypotheses” and argued that the result is that “science becomes an automated, blind search for mindless tabular asterisks using thoughtless hypotheses” (p. 799).

Some critics have argued that progress in psychology has been impeded by the use of NHST as it is conventionally done or even that such testing should be banned (Carver, 1993; Hubbard, Parsa, & Luthy, 1997; Hunter, 1997; Loftus, 1991, 1995, 1996; Schmidt, 1992, 1996). A comment by Carver (1978) represents this sentiment: “[NHST] is not only useless, it is also harmful because it is interpreted to mean something it is not” (p. 392). Shaver (1993) saw the dominance of NHST as dysfunctional “because such tests do not provide the information that many researchers assume they do” and argued that such testing “diverts attention and energy from more appropriate strategies, such as replication and consideration of the practical or theoretical significance of results” (p. 294). Cohen (1994) took the position that NHST “has not only failed to support and advance psychology as a science but also has seriously impeded it” (p. 997). Schmidt and Hunter (1997) stated bluntly, “Logically and conceptually, the use of statistical significance testing in the analysis of research data has been thoroughly discredited,” and again, “Statistical significance testing retards the growth of scientific knowledge; it never makes a positive contribution” (p. 37).

Despite the many objections and the fact that they have been raised by numerous writers over many years, NHST has remained a favored—perhaps the favorite—tool in the behavioral and social scientist’s kit (Carver, 1993; Johnstone, 1986). There is little evidence that the many criticisms that have been leveled at the technique have reduced its popularity among researchers. Inspection of a randomly selected issue of the Journal of Applied Psychology for each year from its inception in 1917 through 1994 revealed that the percentage of articles that used significance tests rose from an average of about 17 between 1917 and 1929 to about 94 during the early 1990s (Hubbard et al., 1997). The technique has its defenders, whose positions are considered in a subsequent section of this review, but even many of the critics of NHST have used it in empirical studies after publishing critiques of it (Greenwald, Gonzalez, Harris, & Guthrie, 1996). The persisting popularity of the approach begs an explanation (Abelson, 1997a, 1997b; Falk & Greenbaum, 1995; Greenwald et al., 1996). As Rindskopf (1997) has said, “Given the many attacks on it, null hypothesis testing should be dead” (p. 319); but, as is clear from to the most casual observer, it is far from that.

Several factors have been proposed as contributors
to the apparent imperviousness of NHST to criticism. Among them are lack of understanding of the logic of NHST or confusion regarding conditional probabilities (Berkson, 1942; Carver, 1978; Falk & Greenbaum, 1995), the appeal of formalism and the appearance of objectivity (Greenwald et al., 1996; Stevens, 1968), the need to cope with the threat of chance (Falk & Greenbaum, 1995), and the deep entrenchment of the approach within the field, as evidenced in the behavior of advisors, editors, and researchers (Eysenck, 1960; Rosnow & Rosenthal, 1989b). A great appeal of NHST is that it appears to provide the user with a straightforward, relatively simple method for extracting information from noisy data. Hubbard et al. (1997) put it this way:

> From the researcher’s (and possibly journal editor’s and reviewer’s) perspective, the use of significance tests offers the prospect of effortless, cut-and-dried decision-making concerning the viability of a hypothesis. The role of informed judgment and intimate familiarity with the data is largely superseded by rules of thumb with dichotomous, accept-reject outcomes. Decisions based on tests of significance certainly make life easier. (p. 550)

In the following two major sections of this article, I focus on specific criticisms that have been leveled against NHST. I first consider misconceptions and false beliefs said to be common, and then turn to other criticisms that have been made. With respect to each of the false beliefs, I state what it is, review what various writers have said about it, and venture an opinion as to how serious the problem is. Subsequent major sections deal with defenses of NHST and recommendations regarding its use, proposed alternatives or supplements to NHST, and related recommendations that have been made.

Misconceptions Associated With NHST

Of the numerous criticisms that have been made of NHST or of one or another aspect of ways in which it is commonly done, perhaps the most pervasive and compelling is that NHST is not well-understood by many of the people who use it and that, as a consequence, people draw conclusions on the basis of tests of significance that the data do not justify. Although most research psychologists use statistics to help them interpret experimental findings, it seems safe to assume that many who do so have not had a lot of exposure to the mathematics on which NHST is built. It may also be that the majority are not highly acquainted with the history of the development of the various approaches to statistical evaluation of data that are widely used and with the controversial nature of the interactions among some of the primary developers of these approaches (Gigerenzer & Murray, 1987). Rozeboom (1960) suggested that experimentalists who have specialized along lines other than statistics are likely to unquestioningly apply procedures learned by rote from persons assumed to be more knowledgeable of statistics than they. If this is true, it should not be surprising to discover that many users of statistical tests entertain misunderstandings about some aspects of the tests they use and of what the outcomes of their testing mean.

It is not the case, however, that all disagreements regarding NHST can be attributed to lack of training or sophistication in statistics; experts are not of one mind on the matter, and their differing opinions on many of the issues help fuel the ongoing debate. The presumed commonness of specific misunderstandings or misinterpretations of NHST, even among statistical experts and authors of books on statistics, has been noted as a reason to question its general utility for the field (Cohen, 1994; McMan, 1995; Tryon, 1998).

There appear to be many false beliefs about NHST. Evidence that these beliefs are widespread among researchers is abundant in the literature. In some cases, what I am calling a false belief would be true, or approximately so, under certain conditions. In those cases, I try to point out the necessary conditions. To the extent that one is willing to assume that the essential conditions prevail in specific instances, an otherwise-false belief may be justified.

**Belief That p is the Probability That the Null Hypothesis Is True and That 1−p Is the Probability That the Alternative Hypothesis Is True**

Of all false beliefs about NHST, this one is arguably the most pervasive and most widely criticized. For this reason, it receives the greatest emphasis in the present article. Contrary to what many researchers appear to believe, the value of \( p \) obtained from a null hypothesis statistical test is not the probability that \( H_0 \) is true; to reject the null hypothesis at a confidence level of, say, .05 is not to say that given the data the probability that the null hypothesis is true is .05 or less. Furthermore, inasmuch as \( p \) does not represent the probability that the null hypothesis is true, its complement is not the probability that the alternative hypothesis, \( H_A \), is true. This has been pointed out many times (Bakan, 1966; Berger & Sellke, 1987;

The value of $p$ is the probability of obtaining a value of a test statistic, say, $D$, as large as the one obtained—conditional on the null hypothesis being true—$p(D \mid H_0)$: which is not the same as the probability that the null hypothesis is true, conditional on the observed result, $p(H_0 \mid D)$. As Falk (1998b) pointed out, $p(D \mid H_0)$ and $p(H_0 \mid D)$ can be equal, but only under rare mathematical conditions. To borrow Carver’s (1978) description of NHST.

Figuring out how likely a difference of a given size is when the hypothesis of no difference is true is not the same as figuring out how likely it is that the hypothesis is true when a difference of a given size is observed.

A clear distinction between $p(D \mid H_0)$ and $p(H_0 \mid D)$, or between $p(D \mid H)$ and $p(H \mid D)$ more generally, appears to be one that many people fail to make (Bar-Hillel, 1974; Berger & Berry, 1988; Birnbaum, 1982; Dawes, 1988; Dawes, Mirels, Gold, & Donahue, 1993; Kahneman & Tversky, 1973). The tendency to see these two conditional probabilities as equivalent, which Dawes (1988) referred to as the “confusion of the inverse,” bears some resemblance to the widely noted “premise conversion error” in conditional logic, according to which $\text{If } P \text{ then } Q$ is erroneously seen as equivalent to $\text{If } Q \text{ then } P$ (Henle, 1962; Revlis, 1975). Various explanations of the premise conversion error have been proposed. A review of them is beyond the scope of this article.

Belief that $p$ is the probability that the null hypothesis is true (the probability that the results of the experiment were due to chance) and that $1-p$ represents the probability that the alternative hypothesis is true (the probability that the effect that has been observed is not due to chance) appears to be fairly common, even among behavioral and social scientists of some eminence. Gigerenzer (1993), Cohen (1994), and Falk and Greenbaum (1995) have given examples from the literature. Even Fisher, on occasion, spoke as though $p$ were the probability that the null hypothesis is true (Gigerenzer, 1993).

Falk and Greenbaum (1995) illustrated the illusory nature of this belief with an example provided by PAuker and PAuker (1979):

For young women of age 30 the incidence of live-born infants with Down’s syndrome is 1/885, and the majority of pregnancies are normal. Even if the two conditional probabilities of a correct test result, given either an affected or a normal fetus, were 99.5 percent, the probability of an affected child, given a positive test result, would be only 18 percent. This can be easily verified using Bayes’ theorem. Thus, if we substitute “The fetus is normal” for $H_0$ and “The test result is positive (i.e. indicating Down’s syndrome)” for $D$, we have $p(D \mid H_0) = .005$, which means $D$ is a significant result, while $p(H_0 \mid D) = .82$ (i.e., 1-.18). (p. 78)

A similar outcome, yielding a high posterior probability of $H_0$ despite a result that has very low probability assuming the null hypothesis, could be obtained in any situation in which the prior probability of $H_0$ is very high, which is often the case for medical screening for low-incidence illnesses. Evidence that physicians easily misinterpret the statistical implications of the results of diagnostic tests involving low-incidence disease has been reported by several investigators (Cassells, Schoenberger, & Graboys, 1978; Eddy, 1982; Gigerenzer, Hoffrage, & Ebert, 1998).

The point of Falk and Greenbaum’s (1995) illustration is that, unlike $p(H_0 \mid D)$, $p$ is not affected by prior values of $p(H_0)$; it does not take base rates or other indications of the prior probability of the null (or alternative) hypothesis into account. If the prior probability of the null hypothesis is extremely high, even a very small $p$ is unlikely to justify rejecting it. This is seen from consideration of the Bayesian equation for computing a posterior probability:

$$p(H_0 \mid D) = \frac{p(D \mid H_0)p(H_0)}{p(D \mid H_0)p(H_0) + p(D \mid H_A)p(H_A)} \quad (1)$$

Inasmuch as $p(H_A) = 1 - p(H_0)$, it is clear from this equation that $p(H_0 \mid D)$ increases with $p(H_0)$ for fixed values of $p(D \mid H_0)$ and $p(D \mid H_A)$ and that as $p(H_0)$ approaches 1, so does $p(H_0 \mid D)$.

A counter to this line of argument might be that
situations like those represented by the example, in which the prior probability of the null hypothesis is very high (in the case of the example 884/885), are special and that situations in which the prior probability of the null is relatively small are more representative of those in which NHST is generally used. Cohen (1994) used an example with a similarly high prior \( p(H_0) \) — probability of a random person having schizophrenia — and was criticized on the grounds that such high prior probabilities are not characteristic of those of null hypotheses in psychological experiments (Baril & Cannon, 1995; McGraw, 1995). Falk and Greenbaum (1995) contended, however, that the fact that one can find situations in which a small value of \( p(D \mid H_0) \) does not mean that the posterior probability of the null, \( p(H_0 \mid D) \), is correspondingly small discredits the logic of tests of significance in principle.

Their general assessment of the merits of NHST is decidedly negative. Such tests, they argued, “fail to give us the information we need but they induce the illusion that we have it” (p. 94). What the null hypothesis test answers is a question that we never ask: What is the probability of getting an outcome as extreme as the one obtained if the null hypothesis is true?

None of the meaningful questions in drawing conclusions from research results — such as how probable are the hypotheses? How reliable are the results? What is the size and impact of the effect that was found? — is answered by the test. (Falk & Greenbaum, 1995, p. 94)

Berger and Sellke (1987) have shown that, even given a prior probability of \( H_0 \) as large as .5 and several plausible assumptions about how the variable of interest (\( D \) in present notation) is distributed, \( p \) is invariably smaller than \( p(H_0 \mid D) \) and can differ from it by a large amount. For the distributions considered by Berger and Sellke, the value of \( p(H_0 \mid D) \) for \( p = .05 \) varies between .128 and .290; for \( p = .001 \), it varies between .0044 and .0088. The implication of this analysis is that \( p = .05 \) can be evidence, but weaker evidence than generally supposed, of the falsity of \( H_0 \) or the truth of \( H_A \). Similar arguments have been made by others, including Edwards, Lindman, and Savage (1963), Dickey (1973, 1977), and Lindley (1993). Edwards et al. stressed the weakness of the evidence that a small \( p \) provides, and they took the position that "a \( t \) of 2 or 3 may not be evidence against the null hypothesis at all, and seldom if ever justifies much new confidence in the alternative hypothesis” (p. 231).

A striking illustration of the fact that \( p \) is not the probability that the null hypothesis is true is seen in what is widely known as Lindley’s paradox. Lindley (1957) described a situation to demonstrate that

if \( H \) is a simple hypothesis and \( x \) the result of an experiment, the following two phenomena can occur simultaneously: (i) a significance test for \( H \) reveals that \( x \) is significant at, say, the 5% level; (ii) the posterior probability of \( H \), given \( x \), is, for quite small prior probabilities of \( H \), as high as 95% (p. 187)

Although the possible coexistence of these two phenomena is usually referred to as Lindley’s paradox, Lindley (1957) credited Jeffreys (1937/1961) as the first to point it out, but Jeffreys did not refer to it as a paradox. Others have shown that for any value of \( p \), no matter how small, a situation can be defined for which a Bayesian analysis would show the probability of the null to be essentially 1. Edwards (1965) described the situation in terms of likelihood ratios (discussed further below) this way:

Name any likelihood ratio in favor of the null hypothesis, no matter how large, and any significance level, no matter how small. Data can always be invented that will simultaneously favor the null hypothesis by at least that likelihood ratio and lead to rejection of that hypothesis at at least that significance level. In other words, data can always be invented that highly favor the null hypothesis, but lead to its rejection by an appropriate classical test at any specified significance level. (p. 401)

The condition under which the two phenomena mentioned by Lindley (1957) can occur simultaneously is that one’s prior probability for \( H \) be concentrated within a narrow interval and one’s remaining prior probability for the alternative hypothesis be relatively uniformly distributed over a large interval. In terms of the notation used in this article, the probability distributions involved are those of \( (D \mid H_0) \) and \( (D \mid H_A) \), and the condition is that the probability distribution of \( (D \mid H_0) \) be concentrated whereas that of \( (D \mid H_A) \) be diffuse.

Lindley’s paradox recognizes the possibility of \( p(H_0 \mid D) \) being large (arbitrarily close to 1) even when \( p(D \mid H_0) \) is very small (arbitrarily close to 0). For present purposes, what needs to be seen is that it is possible for \( p(D \mid H_A) \) to be smaller than \( p(D \mid H_0) \) even when \( p(D \mid H_0) \) is very small, say, less than .05. We should note, too, that Lindley’s “paradox” is paradoxical only to the degree that one assumes that a small \( p(D \mid H) \) is necessarily indicative of a small \( p(H \mid D) \).
Perhaps the situation can be made clear with a related problem. Imagine two coins, one of which, $F$, is fair in the sense that the probability that it will come up heads when tossed, $p_F$, is constant at $0.5$ and the other of which, $B$, is biased in that the probability that it will come up heads when tossed, $p_B$, is constant at some value other than $0.5$. Suppose that one of the coins has been tossed $n$ times, yielding $k$ heads and $n-k$ tails, and that our task is to tell which of the two coins is more likely to have been the one tossed.

The probability of getting exactly $k$ heads in $n$ tosses given the probability $p$ of heads on each toss is the binomial

$$\binom{n}{k} p^k (1-p)^{n-k},$$

where

$$\binom{n}{k}$$

denotes the number of combinations of $n$ things taken $k$ at a time. Suppose, for example, that the coin was tossed 100 times and yielded 60 heads and 40 tails. Letting $p(100D_{60} \mid H_F)$ represent the probability of getting this outcome with a fair coin, we have

$$p(100D_{60} \mid H_F) = \binom{100}{60}(0.5)^{60}(1-0.5)^{40} \approx 0.011.$$  

The probability that 100 tosses of a fair coin would yield 60 or more heads is

$$p(100D_{\geq 60} \mid H_F) = \sum_{k=60}^{100} p(100D_k \mid H_F) \approx 0.028.$$  

Thus, by the conventions of NHST, one would have a result that would permit rejection of the null hypothesis at the .05 level of significance with a one-tailed test.

The Bayesian approach to this problem is to compare the posterior odds ratio, which takes into account for each coin the probability that it would produce the observed outcome (60 heads in 100 tosses) if selected and the probability of it being selected:

$$\frac{p(H_F)_{\text{post}}}{p(H_B)_{\text{post}}} = \frac{p(100D_{60} \mid H_F) p(H_F)_{\text{prior}}}{p(100D_{60} \mid H_B) p(H_B)_{\text{prior}}}.$$

The ratio to the left of the equal sign is the posterior odds ratio (expressed in this case as the odds favoring $H_F$) and is usually represented as $\Omega_{\text{post}}$. The ratio of conditional probabilities,

$$\frac{p(100D_{60} \mid H_F)}{p(100D_{60} \mid H_B)},$$

is referred to as the Bayes factor or the likelihood ratio and is commonly denoted by $\lambda$; the ratio of the two prior probabilities,

$$\frac{p(H_F)_{\text{prior}}}{p(H_B)_{\text{prior}}}.$$

is the prior odds and may be represented as $\Omega_{\text{prior}}$. So the equation of interest is more simply expressed as $\Omega_{\text{post}} = \lambda \Omega_{\text{prior}}$. The posterior odds, the odds in view of the data, are simply the prior odds multiplied by the Bayes factor. If the prior odds ratio favoring one hypothesis over the other is very large, even a large Bayes factor in the opposite direction may not suffice to reverse the direction of the balance of evidence, and if only the Bayes factor is considered, this fact will not be apparent. (This is the point of Falk and Greenbaum's, 1995, illustration with Down's syndrome births.)

In our coin-tossing illustration, we are assuming that the coins had equal probability of being selected, so the prior odds ratio is 1 and the posterior odds ratio is simply the Bayes factor or likelihood ratio:

$$\Omega_{\text{post}} = \lambda = \frac{p(100D_{60} \mid H_F)}{p(100D_{60} \mid H_B)} = \frac{100C_{60}(0.5)^{60}(1-0.5)^{40}}{100C_{60}(0.4)^{60}(1-0.4)^{40}} \approx 0.011.$$

We have already found the numerator of this ratio to be approximately .011, but to determine the value of $\lambda$ we also need to be able to compute the denominator of the likelihood ratio. If we knew the value of $p_B$, the probability of heads for the biased coin, we could compute this easily. Suppose, for example, we knew $p_B$ to be .6. Then we would have

$$p(100D_{60} \mid H_B) = \binom{100}{60}(0.6)^{60}(1-0.6)^{40} \approx 0.081.$$  

So, for a coin with a bias of .6 for heads and an outcome of 60 heads in 100 tosses, the likelihood ratio (dis)favoring $H_F$ is approximately .010844/.081219—about .134, or better than a 7:1 ratio in favor of $H_B$.

However, suppose the biased coin were biased for tails, with, say, a probability of .4 of producing heads. In this case the probability of an outcome of 60 heads given the biased coin would be

$$p(100D_{60} \mid H_B) = \binom{100}{60}(0.4)^{60}(1-0.4)^{40} \approx 2.44249 \times 10^{-5},$$

The Bayesian approach to this problem is to compare the posterior odds ratio, which takes into account for each coin the probability that it would produce the observed outcome (60 heads in 100 tosses) if selected and the probability of it being selected:

$$\frac{p(H_F)_{\text{post}}}{p(H_B)_{\text{post}}} = \frac{p(100D_{60} \mid H_F) p(H_F)_{\text{prior}}}{p(100D_{60} \mid H_B) p(H_B)_{\text{prior}}}.$$
Suppose we know nothing about \( p \). To be able to compute \( \lambda \) we would have to assume something. We could make any assumption we want but would have to recognize that any conclusions drawn would be valid only if that assumption were true. In the absence of any basis for believing otherwise, one might feel that the default assumption should be that all possible biases are equally likely. Inasmuch as there are infinitely many possibilities, the probability associated with any one of them must be infinitely small. However, for purposes of this illustration, let us assume that only integer multiples of \( .01 \) are allowed; that is, the bias could be \(.34, .55, .74, \) but not \(.342, .551, \) or \(.7436 \). So, disallowing 0 and 1 (to preclude having to divide by 0) and reserving \(.50 \) for the fair coin, we allow 98 different biases, each with equal probability. (It will simplify things and do no violence to the discussion if we approximate this probability with \(.01 \).)

The situation is analogous to one in which we know that the coin that was tossed was either (a) with probability \(.50 \), the fair coin, or (b) with probability \(.50 \), a randomly selected 1 of 98 coins, each with a different bias equal to some integer multiple of \(.01 \) between \(.01 \) and \(.99 \), excluding \(.50 \). The prior probability of selection of the fair coin is \(.50 \), and that of the selection of a specific biased coin is \(.50 \times (1/98), \) or approximately \(.005 \). The posterior probability that the coin selected was the fair one given the outcome \( D \) is

\[
\frac{p(H_F | D)}{p(D | H_F)p(H_F) + p(D | H_B)p(H_B)}
\]

where

\[
p(D | H_B)p(H_B) = \sum_{i=1}^{49} p(D | H_{B_i})p(H_{B_i}) + \sum_{i=51}^{99} p(D | H_{B_i})p(H_{B_i})
\]

and \( p_{B_i} = .01i \).

Inasmuch as \( p(H_F) = .5 \) and \( p(H_B) = .005 \) for all \( i \), we can write

\[
p(H_F | D) = \frac{p(D | H_F)p(H_F)}{p(D | H_F)p(H_F) + p(D | H_B)p(H_B)}
\]

Table 2 shows \( \lambda \) for values of \( p_B \) ranging from \(.05 \) to \(.95 \), given 60 heads in 100 tosses. (The middle column is \( p_{(100|H_B)} \), the probability of getting 60 heads in 100 tosses given the associated bias for heads indicated in the left column.) \( H_B \) is favored over \( H_F \) (i.e., \( \lambda < 1 \)) only when the bias on \( H_B \) is between slightly more than \(.5 \) and a little less than \(.7 \). Inasmuch as we are assuming that the fair and biased coins are equally likely to have been tossed, the likelihood ratio is also the posterior odds ratio: so, given an outcome of 60 heads in 100 tosses, the posterior odds ratio favors \( H_B \) over \( H_F \) for a bias for heads between about \(.5 \) and \(.7 \), and it favors \( H_B \) over \( H_F \) otherwise.

Suppose we know nothing about \( p_B \). To be able to compute \( \lambda \) we would have to assume something. We could make any assumption we want but would have

<table>
<thead>
<tr>
<th>( p_B )</th>
<th>( (100)^{p_B}(1-p_B)^{40} )</th>
<th>( \lambda )</th>
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<td>.15</td>
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<tr>
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<td>2.11 ( \times 10^{-18} )</td>
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<tr>
<td>.25</td>
<td>1.37 ( \times 10^{-14} )</td>
<td>7.89 ( \times 10^{11} )</td>
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<tr>
<td>.30</td>
<td>3.71 ( \times 10^{-10} )</td>
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<td>.35</td>
<td>1.99 ( \times 10^{-7} )</td>
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<td>2.44 ( \times 10^{-5} )</td>
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<td>2.32 ( \times 10^{-6} )</td>
<td>4.68 ( \times 10^{6} )</td>
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<tr>
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<td>8.85 ( \times 10^{-10} )</td>
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</tr>
<tr>
<td>.95</td>
<td>5.76 ( \times 10^{-26} )</td>
<td>1.88 ( \times 10^{23} )</td>
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Note. \( \lambda = p_{(100|H_F)}p_{(100|H_B)} \). F = fair; B = biased.
If we apply this equation to the case of 60 heads in 100 tosses, we get \( p(H_F \mid 100 \ D_{60}) = .525 \) and its complement, \( p(H_B \mid 100 \ D_{60}) = .475 \), which makes the posterior odds in favor of \( H_F \) 1.11.

What the coin-tossing illustration has demonstrated can be summarized as follows. An outcome—60 heads in 100 tosses—that would be judged by the conventions of NHST to be significantly different (\( p < .05 \)) from what would be produced by a fair coin would be considered by a Bayesian analysis either more or less likely to have been produced by the fair coin than by the biased one, depending on the specifics of the assumed bias. In particular, the Bayesian analysis showed that the outcome would be judged more likely to have come from the biased coin only if the bias for heads were assumed to be greater than .5 and less than .7. If the bias were assumed equally likely to be anything (to the nearest hundredth) between .01 and .99 inclusive, the outcome would be judged to be slightly more likely to have been produced by the fair coin than by the biased one. These results do not depend on the prior probability of the fair coin being smaller than that of the biased one. Whether it makes sense to assume that all possible biases are equally likely is a separate question. Undoubtedly, alternative assumptions would be more reasonable in specific instances. In any case, the posterior probability of \( H_F \) can be computed only if whatever is assumed about the bias is made explicit. Other discussions of the possibility of relatively large \( p(H_0 \mid D) \) in conjunction with relatively small \( p(D \mid H_0) \) may be found in Edwards (1965), Edwards et al. (1963), I. J. Good (1956, 1981/1983b), and Shafer (1982).

Comment. The belief that \( p \) is the probability that the null hypothesis is true is unquestionably false. However, as Berger and Sellke (1987) have pointed out,

\[ p(H_0 \mid D) \]

like it or not, people do hypothesis testing to obtain evidence as to whether or not the hypotheses are true, and it is hard to fault the vast majority of nonspecialists for assuming that, if \( p = .05 \), then \( H_0 \) is very likely wrong. This is especially so since we know of no elementary textbooks that teach that \( p = .05 \) is at best very weak evidence against \( H_0 \). (p. 114)

Even to many specialists, I suspect, it seems natural when one obtains a small value of \( p \) from a statistical significance test to conclude that the probability that the null hypothesis is true must also be very small. If a small value of \( p \) does not provide a basis for this conclusion, what is the purpose of doing a statistical significance test? Some would say the answer is that such tests have no legitimate purpose.

This seems a harsh judgment, especially in view of the fact that generations of very competent researchers have held, and acted on, the belief that a small value of \( p \) is good evidence that the null hypothesis is false. Of course, the fact that a judgment is harsh does not make it unjustified, and the fact that a belief has been held by many people does not make it true. However, one is led to ask, Is there any justification for the belief that a small \( p \) is evidence that the null is unlikely to be true? I believe there usually is but that the justification involves some assumptions that, although usually reasonable, are seldom made explicit.

Suppose one has done an experiment and obtained a difference between two means that, according to a \( t \) test, is statistically significant at \( p < .05 \). If the experimental procedure and data are consistent with the assumptions underlying use of the \( t \) test, one is now in a position to conclude that the probability that a chance process would produce a difference like this is less than .05, which is to say that if two random samples were drawn from the same normal distribution the chance of getting a difference between means as large as the one obtained is less than 1 in 20. What one wants to conclude, however, is that the result obtained probably was not due to chance.

As we have noted, from a Bayesian perspective assessing the probability of the null hypothesis contingent on the acquisition of some data, \( p(H_0 \mid D) \), requires the updating of the prior probability of the hypothesis (the probability of the hypothesis before the acquisition of the data). To do that, one needs the values of \( p(D \mid H_0), p(D \mid H_A), p(H_0), \) and \( p(H_A) \). However, the only term of the Bayesian equation that one has in hand, having done NHST, is \( p(D \mid H_0) \). If one is to proceed in the absence of knowledge of the values of the other terms, one must do so on the basis of assumptions, and the question becomes what, if anything, it might be reasonable to assume.

Berger and Sellke (1987) have argued that letting \( p(H_0) \) be less than .5 would rarely be justified: “Who, after all, would be convinced by the statement ‘I conducted a Bayesian test of \( H_0 \), assigning a prior probability of .1 to \( H_0 \), and my conclusion is that \( H_0 \) has posterior probability .05 and should be rejected’” (p. 115). I interpret this argument to mean that even if one really believes the null hypothesis to be false—as I assume most researchers do—one should give it at least equal prior standing with the alternative hypothesis as a matter of conservatism in evidence evalua-
tion. One can also argue that in the absence of compelling reasons for some other assumption, the default assumption should be that \( p(H_0) = p(H_\lambda) = .5 \) and that \( p(D \mid H_0) = .05 \), so we can write

\[
p(H_0 \mid D) = \frac{.05}{.05 + p(D \mid H_\lambda)}.
\]

From this it is clear that with the stated supposition, \( p(H_0 \mid D) \) varies inversely with \( p(D \mid H_\lambda) \), the former going from 1 to approximately .048 as the latter goes from 0 to 1.

Table 3 shows \( p(H_0 \mid D) \) for values of \( p(D \mid H_\lambda) \) ranging from 0 to 1 for \( p(D \mid H_0) = .05 \) (left column), .01 (center column), and .001 (right column). As is clear from the table, increasing or decreasing the value of \( p(D \mid H_\lambda) \) while holding everything else constant changes the value of \( p(H_0 \mid D) \) in the opposite direction. In general, the larger the value of \( p(D \mid H_\lambda) \), the better proxy \( p(D \mid H_0) \) is for \( p(H_0 \mid D) \). For \( p(D \mid H_\lambda) = .5 \), \( p(H_0 \mid D) \) is about twice the value of \( p(D \mid H_0) \). Even for relatively small values of \( p(D \mid H_\lambda) \), a \( p(D \mid H_0) \) of .01 or .001 represents fairly strong evidence against the null: For example, if \( p(D \mid H_\lambda) = .2 \), a \( p(D \mid H_0) \) of .01 is equivalent to a \( p(D \mid H_0) \) of .048 and a \( p(D \mid H_0) \) of .001 is equivalent to a \( p(D \mid H_0) \) of .0050.

In short, although \( p(D \mid H_\lambda) \) is not equivalent to \( p(D \mid H_0) \), if one can assume that \( p(H_\lambda) \) is at least as large as \( p(H_0) \) and that \( p(D \mid H_\lambda) \) is much larger than \( p(D \mid H_0) \), then a small value of \( p \), that is, a small value of \( p(D \mid H_\lambda) \), can be taken as a proxy for a relatively small value of \( p(H_0 \mid D) \). There are undoubtedly exceptions, but the above assumptions seem appropriate to apply in many, if not most, cases. (For related analyses, see Baril & Cannon, 1995, and McGraw, 1995.) Usually one does not do an experiment unless one believes there to be a good chance that one's hypothesis is correct or approximately so, or at least that the null hypothesis is very probably wrong. Also, at least for the majority of experiments that are published, it seems reasonable to suppose that the results that are reported are considered by the experimenter to be much more likely under the alternative hypothesis than under the null. The importance of the latter assumption is recognized in the definition of null hypothesis given by James and James (1959) quoted at the beginning of this article, which explicitly describes evidence as unfavorable to a null hypothesis "if the random sample has a low probability under the null hypothesis and a higher one under some admissible alternative hypothesis" (p. 195, emphasis added). DeGroot (1973) made the same point in noting, in effect, that a small \( p \) may be considered strong evidence against \( H_0 \) presumably because one has in mind one or more alternative hypotheses for which the obtained result is much more probable than it is under the null.

In a defense of the use of classical statistics for hypothesis evaluation, W. Wilson, Miller, and Lower (1967) conceded that,

under special conditions, including presumably a specifiable alternative distribution, blind use of a classical analysis might result in a rejection of the null when a defensible Bayesian analysis, considering only the specifiable alternative, might show that the data actually support the null. (p. 192)

They were quick to add that they know of no real-life instance in which this has been demonstrated. It is also important to note that with all the distributions of \( D \) considered by Berger and Sellke (1987), \( p(H_0 \mid D) \) varies monotonically with \( p \); so the smaller the value of \( p \), the stronger the evidence against \( H_0 \) and for \( H_\lambda \). As a general rule, a small \( p \), say, \( p < .001 \), is reasonably strong evidence against \( H_0 \) but not as strong as is usually assumed. Lindley (1993) also made the point that the significance level is typically smaller than the posterior probability of the null hypothesis as calculated with Bayes's rule and that if a small value, say, .05, suffices to cast doubt on the null, "it follows that null hypotheses will be more easily discounted

<table>
<thead>
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<tr>
<td>1.0</td>
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<td>.5010</td>
<td>.50100</td>
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</tbody>
</table>

Note. The prior probabilities, \( p(H_0) \) and \( p(H_\lambda) \), are assumed to be .5.
using Fisher’s method rather than the Bayesian approach” (p. 25). (This puts Melton’s, 1962, well-publicized refusal to publish results with \( p < .05 \) while editor of the Journal of Experimental Psychology in a somewhat more favorable light than do some of the comments of his many critics.)

The numbers in Table 3 represent the condition in which \( p(H_0) = p(H_A) = .5 \). We should note that when everything else is held constant, \( p(H_0 \mid D) \) varies directly with \( p(H_A) \) and of course inversely with \( p(H_A) \). We can also look at the situation in terms of the Bayes factor or likelihood ratio (I. J. Good, 1981/1983b) and ask not what the probability is of either \( H_A \) or \( H_0 \) view of the data, but which of the two hypotheses the data favor. This approach does not require any knowledge or assumptions about \( p(H_A) \) or \( p(H_0) \), but it does require knowledge, or an estimate, of the probability of the obtained result, conditional on the alternative hypothesis, \( p(D \mid H_A) \). Whenever the probability of a result conditional on the alternative hypothesis is greater than the probability of the result conditional on the null, \( \lambda > 1 \), the alternative hypothesis gains support. The strength of the support is indicated by the size of \( \lambda \). (I. J. Good, 1981/1983b, pointed out that the logarithm of this ratio was called the weight of evidence in favor of \( H_A \) by C. S. Peirce, 1878/1956, as well as by himself [I. J. Good, 1950] and others more recently.)

The fact that evaluating hypotheses in terms of the Bayes factor alone does not require specification of the prior probabilities of the hypotheses is an advantage. However, it is also a limitation of the approach inasmuch as it gives one only an indication of the direction and degree of change in the evidence favoring one hypothesis over the other but does not provide an indication of what the relative strengths of the competing hypotheses—in view of the results—are.

In my view, the most important assumption required by the belief that \( p \) can be a reasonable proxy for \( p(H_0 \mid D) \) is that \( p(D \mid H_A) \) is much greater than \( p(D \mid H_0) \). It seems likely that, if asked, most experimenters would say that they make this assumption. But is it a reasonable one? I find it easier to imagine situations in which it is than situations in which it is not. On the other hand, it is not hard to think of cases in which the probability of a given result would be very small under either the null or the alternative hypothesis. One overlooks this possibility when one argues that because the prior probability of a specified event was small, the event, having occurred, must have had a nonchance cause.

Essentially all events, if considered in detail, are low-probability events, and for this reason the fact that a low-probability event has occurred is not good evidence that it was not a chance event. (Imagine that 10 tosses of a coin yielded a tails [T] and heads [H] sequence of TTHHHHTHTH. The probably of getting precisely this sequence given a fair coin, \( p(D \mid chance) \), in 10 consecutive tosses is very small, less than .001. However, it clearly does not follow that the sequence must have been produced by a nonchance process.) I. J. Good (1981/1983b) applied this fact to the problem of hypothesis evaluation this way:

We never reject a hypothesis \( H \) merely because an event \( E \) of very small probability (given \( H \)) has occurred although we often carelessly talk as if that were our reason for rejection. If the result \( E \) of an experiment or observation is described in sufficient detail its probability given \( H \) is nearly always less than say one in a million. (p. 133)

I. J. Good (1981/1983b) quoted Jeffreys (1961) on the same point: “If mere probability of the observation, given the hypothesis, was the criterion, any hypothesis whatever would be rejected” (p. 315). What one needs to know is how the probability of the event in question given the null hypothesis compares with the probability of the event given the alternative hypothesis.

Usually \( p(D \mid H_A) \) is not known—often the exact nature of \( H_A \) is not specified—and sometimes one may have little basis for even making an assumption about it. If one can make an assumption about the value of \( p(D \mid H_A) \), one may have the basis for an inference from \( p \) to \( p(H_0 \mid D) \), or at least from \( p \) to \( \lambda \), that will be valid under that assumption. It is desirable, of course, when inferences that rest on assumptions are made that those assumptions be clearly identified. It must be noted, too, that, in the absence of knowledge, or some assumption, about the value of \( p(D \mid H_A) \), \( p \) does not constitute a reliable basis for making an inference about either \( p(H_0 \mid D) \) or \( \lambda \). We can say, however, that in general with other things being equal, the smaller the value of \( p \), the larger the Bayes factor favoring \( H_A \). The claim that \( p \) is likely to be smaller than \( p(H_0 \mid D) \) is not necessarily an argument against using NHST in principle but only a basis for concluding that a small \( p \) is not as strong evidence against the null hypothesis as its value suggests, and it is obviously a basis for not equating \( p \) with \( p(H_0 \mid D) \).
Belief That Rejection of the Null Hypothesis Establishes the Truth of a Theory That Predicts It to Be False

Sometimes researchers appear to assume that rejection of the null hypothesis is by itself an adequate basis for accepting a theory that implies the null hypothesis is false. The line of reasoning from "the null hypothesis is false" to "the theory is therefore true" involves the logical fallacy of affirming the consequent: "If the theory is true, the null hypothesis will prove to be false. The null hypothesis proved to be false; therefore, the theory must be true"—if then Q; therefore P.

Most researchers would probably agree that rejection of the null hypothesis does not prove a theory that predicts its rejection, but would hold that it constitutes evidence in favor of the theory. Lykken (1968) has challenged the notion that experimental confirmation of a theoretically derived prediction or hypothesis should increase one's confidence in the theory by a nontrivial amount, especially when one's prior confidence is low: "[This rule] is wrong not only in a few exceptional instances but as it is routinely applied in the majority of experimental reports in the psychological literature." (p. 152). Lykken's justification for this position is the claim that predictions in psychology often specify only the direction of a difference or correlation and the assumption that statistically significant differences or correlations are likely to be found for reasons unrelated to the theoretical hypothesis, especially if the sample size is large. In other words, prediction of a directional effect of unspecified size is not very precise, and having the prediction prove to be correct is not very surprising whether the theory from which it was made is true or false.

Lykken (1968) argued for acceptance of the harsh conclusion that a single experimental finding of this usual kind (confirming a directional prediction), no matter how great its statistical significance, will seldom represent a large enough increment of corroboration for the theory from which it was derived to merit very serious scientific attention. (p. 153)

Theory corroboration requires the testing of multiple predictions because the chance of getting statistically significant results for the wrong reasons in any given case is surprisingly high. The finding of statistical significance, Lykken concluded is perhaps the least important attribute of a good experiment; it is never a sufficient condition for concluding that a theory has been corroborated, that a useful empirical fact has been established with reasonable confidence—or that an experiment report ought to be published. (p. 158)

According to this view, even if one interprets statistical significance as evidence against the hypothesis that an observed effect was due to chance, statistical significance by itself does not warrant concluding that a specific nonchance explanation of the effect is true. The latter step also requires ruling out other plausible nonchance explanations (Erwin, 1998; Snow, 1998).

Whether the "nonchance" mechanism producing a research result (i.e., one that yields a nonzero effect) is the one proposed by the investigator can only be determined by good research design—namely the elimination of competing explanations through proper control of potential confounds and a convincing translation of the substantive question into an empirical hypothesis. (Hayes, 1998, p. 203)

Comment. The claim that interpreting the verification of a prediction as supportive of the predicting theory involves committing the logical fallacy of affirming the consequent may be applied to much of theory testing in science generally. The preeminent way of testing any scientific theory is to see whether its predictions prove to be true, and a theory gains credence to the extent that they do. Although the logic has the technically fallacious form of affirming the consequent, it is nevertheless used with great success.

Showing a specific prediction of a theory to be true does not prove the theory of course, but it does add to its credence. How much support the verification of a theory's prediction provides for the theory depends on a variety of factors, such as the relative uniqueness of the prediction to the theory (is it made by competing theories as well?), how surprising the prediction is, the preciseness of the prediction, the degree of correspondence between the prediction and the observation, and so on. An extended discussion of this topic may be found in Polya (1954a, 1954b). The idea of the relative uniqueness of a prediction is found also in the Bayesian notion of diagnosticity: Data are said to be the more diagnostic with respect to competing hypotheses, say, $H_A$ against $H_0$, the larger the ratio of the conditional probabilities of the data given the hypotheses (the likelihood ratio) when the larger of the conditional probabilities is the numerator of the ratio. For
the case of \( p(D \mid H_A) > p(D \mid H_0) \), the diagnosticity of the data is reflected in the size of the ratio

\[
\frac{p(D \mid H_A)}{p(D \mid H_0)}
\]

In light of the widespread use of prediction verification as a method of theory corroboration in science, I see the objection expressed by Lykken (1968) and others as concern that psychologists often take rejection of the null hypothesis to be stronger support for a theory that predicted it than it really is.

Arguing that the corroboration that a theory receives from a predicted fact is weak unless the fact has low prior probability and there are few alternative theories, Meehl (1997) pointed out that "the fact of a nonzero difference or correlation, such as we infer by refuting the null hypothesis, does not have such a low probability because in social science everything correlates with almost everything else, theory aside" (p. 393). Meehl (1997) also stressed the importance of distinguishing clearly between the substantive theory of interest and the statistical hypothesis that is deduced from it, and he contended that it is a distinction that generally is not made: "Hardly any statistics textbooks and, so far as I have been able to find out, hardly any statistics or psychology professors lecturing on this process bother to make that distinction, let alone emphasize it" (p. 401).

Chow (1996, 1998a) has made the distinction sharply in a treatment of NHST that presents the statistical hypothesis as the major premise of the innermost of a nested sequence of conditional syllogisms, beginning with a major premise containing the substantive hypothesis and ending with one containing a statistical hypothesis. In this representation, each syllogism has the invalid form of affirming the antecedent: If \( P \) then \( Q \); \( Q \), therefore \( P \). Chow (1998a) acknowledged the logical invalidity of this form but contended that its use is justified "by virtue of experimental controls" (p. 174).

If I understand Chow's claim, it is that the experimental controls assure that if \( Q \) occurs, \( P \) is the cause of it, which is to say that the controls rule out other possible causes of \( Q \). In other words, given adequate experimental controls, if \( P \) then \( Q \) can be treated more or less as the biconditional \( \text{if-and-only-if} \) \( P \) then \( Q \), which in combination with \( Q \) justifies the conclusion \( P \). To emphasize the tentativeness of this, Chow (1998a) qualified the conclusion drawn from this form by adding "in the interim (by virtue of experimental controls)" (p. 174). Chow (1998a) argued too, that inasmuch as one has control of extraneous variables in experimental studies but not in nonexperimental studies, data from the latter are more ambiguous than data from the former. However, as Erwin (1998) has pointed out, although the "in the interim" qualification may render the syllogism innocent of the charge of affirming the consequent, it does not guarantee validity; furthermore, in the absence of specification of what constitutes adequate experimental control, Chow's formalism does not help one determine when experimental data are supportive of a hypothesis.

All this being said, given the premise if the theory is true, the null hypothesis will prove to be false, evidence that the null hypothesis is false usually constitutes inductive support of the hypothesis that the theory is true, or so it seems to me. How much support falsification of the null hypothesis provides for the theory depends on a variety of factors, just as in the case of prediction verification more generally. However, high confidence in theories is established in the social sciences, as in the physical sciences, as the consequence of converging evidence from many quarters and never by the observation that a single prediction has proved to be true within some statistical criterion of acceptance (Garner, Hake, & Eriksen, 1956); verification of the single prediction can constitute one of the bits of converging evidence.

Meehl (1967, 1990a, 1997) distinguished between strong and weak uses of statistical significance tests in theory appraisal:

The strong use of significance tests requires a strong theory, one capable of entailing a numerical value of the parameter, or a narrow range of theoretically tolerated values, or a specific function form (e.g., parabola) relating observables. Statistically significant deviation from the predicted point value, narrow interval, or curve type acts as a falsifier of the substantive theory. . . . In the weak use, the theory is not powerful enough to make a point prediction or a narrow range prediction; it can say only that there is some nonzero correlation or some nonzero difference and, in almost all cases, to specify its algebraic direction. (1997, p. 407)

According to this distinction, which is essentially the distinction between acceptance-support (AS) and rejection-support (RS) NHST mentioned earlier, rejection of the null hypothesis is taken as evidence against the theory with the strong use and as evidence for the theory with the weak use. What makes the weak use weak is the difficulty of ruling out factors other than the theorized one as possible determinants of statistically significant effects that are obtained.
Meehl (1997) cautioned that both the strong and weak uses are subject to misinterpretation or abuse. The strong use risks rejection of a theory when in fact a significant difference from a prediction could have been due to any of a variety of reasons other than that the theory was incorrect. The weak use is abused when rejection of the null hypothesis is interpreted as powerful support for a weak theory. Moreover, although most researchers would undoubtedly agree that strong theories are much to be preferred over weak ones, Meehl (1997) expressed some reservations about the strong use of NHST:

Even when the theory is so strong as to permit point predictions ... the uncertainty of the auxiliaries, the doubtfulness of the *ceteris paribus* clause, the unreliability of measuring instruments, and so on, leave us wondering just what we should say when what appears to be a strong Popperian test is successfully passed or—even more so—is failed. (p. 411).

Meehl (1997) noted the possibility that, especially in the early stages of theory construction, an outcome that could be taken literally as a falsification of the theory could equally well be seen as an encouraging sign:

The history of science shows that—even for the most powerful of the exact sciences—numerical closeness to a theoretically predicted observational value is commonly taken as corroborative of a strong theory even if, strictly speaking, it is a falsifier because the observed value deviates “significantly” from the value predicted. (p. 411)

It seems there is no escaping the use of judgment in the use and interpretation of statistical significance tests.

**Belief That a Small *p* Is Evidence That the Results Are Replicable**

Often statistical significance is taken as evidence of the replicability (or reliability) of the obtained experimental outcome; a small value of *p* is considered to mean a strong likelihood of getting the same results on another try (Coleman, 1964; Evans, 1985; R. J. Harris, 1997a; Levy, 1967; Melton, 1962; Reaves, 1992; Shaughnessy & Zechmeister, 1994). In some cases, the complement of *p* appears to be interpreted as an indication of the exact probability of replication. Nunnally (1975), for example, has said that statistical significance at the .05 level can be taken to mean that the odds are 95 out of 100 that the observed difference will hold up in future investigations. A survey of academic psychologists by Oakes (1986) revealed that 60% of the participants held essentially this belief.

Carver (1978) referred to this belief as the “replicability or reliability fantasy,” inasmuch as “nothing in the logic of statistics allows a statistically significant result to be interpreted as directly reflecting the probability that the result can be replicated” (p. 386). Several other writers have noted that a small *p* value does not guarantee replicability of experimental results (Bakan, 1966; Falk, 1998b; Falk & Greenbaum, 1995; Giggenzer, 1991; Lykken, 1968; Rosenthal, 1991; Sohn, 1998b; Thompson, 1996), but the belief that it does appears to be very common among psychologists. (Sohn, 1998b, pointed out that even the fact that a hypothesis is true does not guarantee the replication of an experimental finding.)

**Comment.** It is important to distinguish different connotations that replication can have in this context. It can mean getting exactly, or almost exactly, the same effect—direction and size—in a repetition of an experiment in which conditions are as nearly the same as those of the original as they can be made, or it can mean getting a result that will support the same conclusion (reject or nonreject) regardless of the null hypothesis. Finer distinctions can be made within the latter category (Lykken, 1968; Sidman, 1960). A small *p* does not guarantee replicability in either of the two senses mentioned. Definitely, *p* does not represent the complement of the probability that a result will replicate in either of these senses.

An argument can be (and has been) made, however, that a small *p* does constitute a reasonable basis for expectation with respect to the latter sense—that having obtained a statistically significant result, the smaller the value of *p*, the more likely it is that a replication of the experiment would again produce a statistically significant result (Greenwald et al., 1996; Rosnow & Rosenthal, 1989b; Scarr, 1997). Other things equal, in this sense a bet on replicability of a result that yielded *p* < .001 would be safer than a bet on the replicability of a result that yielded *p* < .05. *p* < .001 tells one that the result obtained would be expected less than 1 time in a thousand when the null hypothesis is true, whereas *p* < .05 tells one the result would be expected less than 1 time in 20 when the null is true. The evidence that the result is real (nonchance) is stronger in the former case and thus provides a firmer basis for the expectation of replication (in the sense of another statistically significant result).

Schmidt and Hunter (1997) contended that “reproducibility requires high statistical power. Even if all
other aspects of a study are carried out in a scientifically impeccable manner, the finding of statistical significance in the original study will not replicate consistently if statistical power is low” (p. 44). It needs to be noted, however, that this observation pertains specifically to the statistical power of the experiment that constitutes the replication attempt. It can be argued that the smaller the sample and the smaller the \( \alpha \) of the original experiment, the larger the effect size must have been to yield a significant result, and that the larger the effect size, the more likely it should yield significance on subsequent experimentation. Inasmuch as the effect size in the population does not change as a consequence of experimental procedure, the probability of getting a significant result in a replication study can be increased by increasing the power of that study relative to that of the original by increasing the sample size.

Discussions of replicability usually do not consider the replicability of nonsignificant results. However, it should be noted that if the results of an experiment yield a large \( p \), it seems likely that a repetition of the experiment would again yield a nonsignificant value of \( p \). So, if obtaining nonsignificant results a second time is considered a replication, a case might be made for the claim that a large \( p \) suggests the likelihood of replicability, but in this case of a nonsignificant result. Whether nonsignificance tends to replicate more consistently than significance is an empirical question; Schmidt and Hunter (1997) have suggested that it does not.

Belief That a Small Value of \( p \) Means a Treatment Effect of Large Magnitude

It is difficult to know how common this belief is. My guess is that, if asked, most researchers would judge it to be false, but this is just a guess. In reporting results of experiments, however, researchers often use language that lends itself to this type of misinterpretation. Reference to effects as “significant” rather than as “statistically significant” invites such misinterpretation, unless the context makes it clear that the latter connotation is intended; and often the context does not rule out the less restrictive meaning.

Other ways of qualifying “significant”—“extremely,” “barely,” “marginally”—can also convey inappropriate meanings. For example, I recently reviewed a manuscript that described a response time that was about 16% slower than another as being “marginally slower” than the latter, because \( .05 < p < .06 \). This is not unusual, in my experience. Undoubtedly, such language often gets modified as a consequence of the editorial review process, but certainly not all of it does; and the fact that it appears in unedited manuscripts indicates the need for greater awareness of the problem.

Comment. The value of \( p \) is not a reliable indication of the magnitude of an effect (Bracey, 1991; Cohen, 1994; Rosenthal, 1993); as Sohn (1998b) has said, “There is no guarantee, from SS [statistical significance], that the mean difference is greater than infinitesimal” (p. 299). On the other hand, \( p \) and effect size are not completely independent, and belief that one can be an indication of the other has some foundation. For fixed sample size and variability, the larger an effect, the smaller that \( p \) is likely to be, and vice versa. The proviso is important, however, because with a large sample or small variability, even a very small effect can prove to be statistically significant (yield a small \( p \)), and with a small sample or large variability even a large effect can fail to attain a conventional level of significance. Parenthetically, we should also note that a large effect is not a guarantee of importance, any more than a small \( p \) value is; although, as a general rule, a large effect seems more likely to be important than a small one, at least from a practical point of view.

Belief That Statistical Significance Means Theoretical or Practical Significance

Confusion between statistical and theoretical or practical significance (or what is sometimes called substantive significance, or in clinical contexts, clinical significance) appears to be a continuing problem, despite warnings against it by many writers over many years (Berkson, 1938, 1942; Cohen, 1965, 1994; Grant, 1962; Gutman, 1977; Hays, 1994; Jacobson, Follette, & Revenstorf, 1984; Rosenthal, 1993; Shaver, 1985, 1993; Tyler, 1931). When one concludes on the basis of a statistical test that the difference between two means is statistically significant, one is saying only that a difference of the observed magnitude is unlikely to be obtained between two samples drawn at random from the same population. Even assuming that one has a basis for going beyond this and concluding that the difference is real—caused by something other than chance—it does not follow that it is either interesting or important. Rosenthal (1983) has argued that because of the
lack of correspondence between statistical and practical significance, investigators who are interested primarily in the practical implications of their results may find NHST to be of limited use.

A special case of failing to distinguish between statistical and substantive significance is the uncritical acceptance of correlations that are significantly different from zero as necessarily worthy of attention. Because there is little reason to expect correlations between uncontrolled variables to be exactly zero in general, it can be argued that testing the hypothesis of no correlation makes little sense in most situations (Cohen, 1994). Abelson (1997a), who also noted that the fact that a correlation is different from zero is not interesting, described the declaring of a reliability coefficient to be nonzero as “the ultimate in stupefyingly vacuous information” (p. 13).

Although it is less frequently discussed than the problem of confusing statistical significance with theoretical or practical significance, there is the opposite problem of confusing lack of statistical significance with lack of theoretical or practical importance. Many would argue that this problem is of little concern, on the grounds that if a result has not been shown to be statistically significant it should be dismissed without further consideration. A major purpose of NHST, the argument goes, is precisely to determine which results are worthy of attempts at causal explanation and which should be dismissed as plausibly due to chance (Gold, 1969; Winch & Campbell, 1969).

Comment. The above may be the majority opinion, but there is an opposing one. Carver (1978) has argued, persuasively in my view, that making statistical significance the criterion for deciding whether to think further about the implications of experimental results is putting the cart before the horse. A better way to proceed is to first consider whether the data at hand are generally supportive of the hypothesis of interest and, only if they are, to consider candidate hypotheses, including the null hypothesis, that might account for them. If one’s research hypothesis predicts a substantial effect of an experimental manipulation on the mean of a specific variable, observation of an effect of negligible size may be considered not sufficiently supportive of the hypothesis to be worth subjecting to a statistical test; whereas it may be worth trying to replicate what appears to be a large effect, as well as testing to see if random variation is among the plausible explanations for it.

Belief That Alpha Is the Probability That if One Has Rejected the Null Hypothesis One Has Made a Type I Error

This belief appears to be very common among psychologists. Oakes (1986) and Pollard and Richardson (1987) have gathered evidence to this effect, and the literature provides many illustrations of it even among outstanding researchers. Grant (1962), for example, claimed that “rejection of \( H_0 \) permits [one] to assert, with a precisely defined risk of being wrong, that the obtained differences were not the product of chance variation” (p. 54). The “precisely defined risk of being wrong” is, I assume, \( \alpha \). If so, the claim, in effect, is that \( \alpha \) is the probability of being wrong in rejecting \( H_0 \). Carver (1978) quoted a similar claim by Hebb (1966). The probability of having made a Type I error, given that one has rejected the null hypothesis, may be represented as \( p(H_0 \mid R_0) \). However, \( \alpha \) is the probability that one will reject the null hypothesis, given that it is true: \( p(R_0 \mid H_0) \); and \( p(R_0 \mid H_0) \) is not the same as \( p(H_0 \mid R_0) \). The confusion between \( p(R_0 \mid H_0) \) and \( p(H_0 \mid R_0) \) is analogous to the confusion between \( p(D \mid H_0) \) and \( p(H_0 \mid D) \).

Anastasi (1988) made a claim that appears to be similar to Grant’s (1962) and Hebb’s (1966) at first glance but is more difficult to understand on more careful consideration:

To say that the difference between two means is significant at the .01 level indicates that we can conclude, with only one chance out of 100 of being wrong, that a difference in the obtained direction would be found if we tested the whole population from which our samples were drawn. (p. 115)

This claim appears to assume that the samples were drawn from the same population, but this is not a reasonable assumption to make in most experimental situations. Further (assuming the two samples were indeed drawn from the same population) it is not clear what is meant here by testing that entire population. If the entire population were measured with respect to the variable of interest, one would have the mean of that variable; there would be no difference about which to be concerned. A claim that is correct is that if one repeatedly took two random samples from the same population and tested the differences between their means, one would expect to get a difference that proved to be significant at the .01 level about one time in 100, but this is quite different from that claim that Anastasi makes.

Falk and Greenbaum (1995; Falk, 1986) have noted
that discussions of Type I error are often couched in language that is sufficiently informal to admit of more than one interpretation. Reference to \( \alpha \) simply as the probability of Type I error, for example, is a case in point. It is not the absolute probability of Type I error, which as noted earlier is the product of \( \alpha \) and the probability that the null is true; nor is it the probability of Type I error conditional on obtaining a \( p \) in the \( \alpha \) region—instead it is the probability of rejecting \( H_0 \) conditional on its being true.

**Belief That the Value at Which Alpha Is Set for a Given Experiment Is the Probability That a Type I Error Will Be Made in Interpreting the Results of That Experiment**

This belief is illustrated by the following comment: “In a directional empirical prediction we can say that 1 or 5% of the time (as we choose) we will be wrong in rejecting the null hypothesis on the basis of such data as these” (W. Wilson & Miller, 1964b, p. 242). Referring to this comment and others by the same authors (W. Wilson & Miller, 1964a), W. Wilson et al. (1967) said, “These writers pointed out that while the probability of rejecting the null hypothesis wrongly is held constant, for example, at the .05 level, the probability of accepting the null hypothesis wrongly varies with the precision of the experiment” (p. 188). Another illustration comes from a comment by Chow (1988) regarding the arbitrariness of the “choice of the alpha level (i.e., the probability of Type-I error)” (p. 105).

The value of \( \alpha \) is the theoretical probability that the null hypothesis will be rejected *if the null hypothesis is true*; it does not guarantee that the probability of rejecting the null hypothesis will be held constant, unless one assumes that the null is always true. In any given experiment, the null hypothesis may be either true or false, and a Type I error is possible only in the former case, so the value of \( \alpha \) sets an upper bound on the probability of a Type I error. If in a typical experiment the null hypothesis is at least as likely to be false as it is to be true, the probability that the experiment will result in the commission of a Type I error will be considerably less than \( \alpha \). It appears that confusion on this point is very common, and not only among researchers who use statistics only occasionally and in cookbook fashion. Pollard and Richardson (1987) quoted five textbooks on experimental design and statistics that equate \( \alpha \) with the probability of making a Type I error.
more than half of all tests performed to be nonsignificant may be summarized as follows: $p(E_1)$ — type I error, $p(E_2)$ — type II error, and $p(E_3)$ — error irrespective of type. Letting $p(H_0)$, $p(E_1)$, $p(E_2)$, and $p(E_3)$ represent, respectively, the absolute probabilities of the null is false, the alternative hypothesis is true, conditional on having rejected it. Sometimes it is taken as the absolute probability of making a Type II error.

**Comment.** $\beta$ is neither of these; it is the probability of failing to reject the null hypothesis, given that it is false. The absolute probability of making a Type II error is the product of $\beta$ and the probability that the null is false, $p(H_0)$. Letting $p(E_1)$, $p(E_2)$, and $p(E_3)$ represent, respectively, the absolute probabilities of Type I error, Type II error, and error irrespective of type, what has been said or implied about these variables may be summarized as follows: $p(E_1) = \alpha p(H_0)$, $p(E_2) = \beta p(H_0)$, and $p(E_3) = p(E_1) + p(E_2)$.

The fact that $\beta$, like many of the other probabilities associated with NHST, is a conditional probability is easily ignored, even by experts on occasion. Schmidt and Hunter (1997), for example, claimed that “with a power of .50, half of all tests in a research literature will be nonsignificant” (p. 40). Recall that power is 1 - $\beta$, so power is .50 only when $\beta = .50$. Power of .50 means that half of all tests with that power performed on samples drawn from different populations will be nonsignificant. The statement would be true of all tests only on the assumption that all tests involve samples drawn from different populations. As I have noted, some writers (including, if I understand their position, Schmidt & Hunter) appear to be willing to make that assumption, but not all are. If we assume that some of the tests are performed on samples drawn from the same population (that the null is sometimes true), then a power of .50 would lead us to expect more than half of all tests performed to be nonsignificant (half of those for which the null was false and well more than half of those for which the null was true). I note that shortly following the statement quoted above, Schmidt and Hunter (1997) made a similar observation, but this time with implicit recognition of its conditionality:

In a research area in which there really is a difference or relation, when the significance test is used to determine whether findings are real or just chance events, the null hypothesis significance test [with power of .5] will provide an erroneous answer about 50% of the time. (p. 40)

**Belief That Failing to Reject the Null Hypothesis Is Equivalent to Demonstrating It to Be True**

Some researchers interpret the absence of a statistically significant effect as strong evidence that the null hypothesis is true (Harcum, 1990; Schmidt, 1996). Furthermore, whether or not they believe that failing to reject the null hypothesis is equivalent to demonstrating it to be true, many researchers make decisions regarding experimental procedures and data analysis as though they believed so. Malgady (1996, 1998) noted that this is often the case in clinical research. In testing the effectiveness of a new drug, for example, failure to reject the hypothesis of no difference between the effect of the drug and that of a placebo may ensure that the drug will not be approved, at least not without further testing. Levin (1993) appeared to take this position in arguing that NHST should precede any discussion of effect sizes: “To talk of effect sizes in the face of results that are not statistically significant does not make sense . . . If it’s not real, call it zero” (p. 379).

One is, in effect, accepting the null hypothesis as true when one takes the failure of $p$ to reach a conventional level of significance as evidence that prior to experimental treatment an experimental group and a control group were equivalent with respect to some measure of interest. The same observation pertains to the common practice of pooling data across specific conditions for subsequent analyses after a test has failed to show the difference between those conditions to be statistically significant.

Failure to reject the null hypothesis could be because the null hypothesis is true; however, in many cases this seems unlikely to be the reason for rejection. If $\alpha$ is set at .05, any $p$ value larger than this constitutes failure to reject the null hypothesis; but one that is close to it, say, .06 or even .10, is hardly compelling evidence that the null hypothesis is true. Moreover, there are many possible reasons for failure to reject the null hypothesis in addition to the null
hypothesis being true (Cook & Campbell, 1979; Lakatos, 1970), faulty experimental design and lack of adequate control of extraneous variables being only two of them; so one is not justified in concluding from failure to reject the null hypothesis that it is true. Statistical significance tests are structured so that the probability that a real effect of a given size will prove to be significant increases with sample size. This being so, it is possible that a result that failed to provide a basis for rejecting the null hypothesis would have succeeded with a larger sample.

The tendency among psychologists to equate failure to reject the null hypothesis with evidence of no difference or of no effect is seen by some critics of NHST as a major deterrent to progress in the field (Hunter, 1997; Schmidt, 1996). More is said on this subject later in the context of discussion of the presumed frequency of Type II error.

Comment. Most psychologists would take the Fisherian position, I believe, that it is never appropriate to speak of accepting the null hypothesis as opposed to failing to reject it (Fisher, 1935). There is the contrary opinion, however, that a conclusion in favor of the null hypothesis can be useful and warranted on some occasions (Binder, 1963; Frick, 1995a; Greenwald, 1975, 1993; Rogers, Howard, & Vessey, 1993; Yeaton & Sechrest, 1986).

Frick (1995a) argued that under certain conditions the null hypothesis should be accepted. The conditions are that (a) it is possible (as many would believe it to be, e.g., if the alternative hypothesis were that people can transmit thoughts by mental telepathy), (b) the results in hand are consistent with it, and (c) the effort to find grounds for rejecting it was a good one. The last condition is reminiscent of Popper's (1959) position that the strongest confirmatory evidence for a scientific hypothesis is failure of concerted efforts of competent researchers to falsify it. Some evidence of a good effort in the context of a psychological experiment, Frick (1995a) suggested, is a confidence interval (which I discuss further below) of small range, because a confidence interval takes into account sample size and variability; thus, the smaller the confidence interval that includes 0, the stronger the evidence for the null hypothesis.

I. J. Good (1981/1983b) argued that if $H_A$ is nonspecific—representing only the complement of a point null hypothesis—compelling evidence of $H_0$ cannot be obtained even if true because there will always be components of $H_A$ that are close enough to be indistinguishable from it. On the other hand, if $H_A$ is expressed as a specific alternative to $H_0$ (not just as its complement) with a specified mean and distribution or a set of means and distributions, then evidence can be obtained to show $H_0$ to be more probable than the alternative. I.J. Good (1981/1983b) noted, too, that if $H_0$ is defined to include a small neighborhood of the point null, evidence favoring it can be obtained even in the absence of assumptions regarding the distribution of $H_A$ or its components. He made an analogy between the null hypothesis and Newtonian mechanics:

If by the truth of Newtonian mechanics we mean that it is approximately true in some appropriate well-defined sense we could obtain strong evidence that it is true; but if we mean by its truth that it is exactly true then it has already been refuted. (p. 135)

Belief That Failure to Reject the Null Hypothesis Is Evidence of a Failed Experiment

The word significant, Eysenck (1960) has argued, "has become a shibboleth which divides the successful from the unsuccessful research" (Eysenck, 1960, p. 269). The tendency to regard failure to reject the null hypothesis as tantamount to having conducted a failed experiment is reinforced by the general reluctance of advisors to accept as good science experiments that did not yield statistically significant results and of editors to publish reports of such experiments.

Comment. Failure to reject the null hypothesis can indeed be the result of a failed experiment in the sense of an experiment that, because of some aspect(s) of its design or implementation, did not yield statistically significant evidence of an important effect that a better-designed or executed experiment would have yielded, but it can also be the result of the absence of any substantive effect to be found.

Misconceptions and Linguistic Ambiguity

It is very easy to use language that reflects one or another of the false beliefs mentioned above. Even experts, including people who have been highly critical of NHST because of the prevalence of misunderstandings of it, sometimes do it. (Examples are given in Gigerenzer, 1993, and Falk & Greenbaum, 1995). Cohen (1994) noted that it is not uncommon to find both correct, $p(D \mid H_0)$, and incorrect, $p(H_0 \mid D)$, interpretations of $p$ in the same textbook. He noted having given the incorrect interpretation himself and being called on it by Oakes (1986).

Consider the following comment by Carver (1978):
Properly interpreted, statistical significance testing provides a $p$ value or the probability of obtaining mean differences of given sizes under the null hypothesis. Thus, the $p$ value may be used to make a decision about accepting or rejecting the idea that chance caused the results. (Carver, 1978, p. 387)

The first of these statements equates $p$ with $p(D \mid H_0)$; the second seems to imply that $p$ can be used to infer the probability that the null is true. As pointed out in the foregoing, without a knowledge of $p(D \mid H_A)$ or of some assumption regarding its value, the value of $p$ does not constitute an adequate basis for inferring whether a particular result was obtained by chance.

The value of $p$ tells us the probability that a chance process would have yielded such a result, but without knowing the probability that a nonchance process would have yielded it and without knowing the prior probabilities, $p(H_0)$ and $p(H_A)$, we do not have what is needed to compute the probability that this particular result was produced by chance.

Falk and Greenbaum (1995) pointed to verbal ambiguity as a major factor contributing to widespread misconceptions about NHST, especially confusions involving conditional probabilities. Linguistic ambiguity and unstated assumptions have been noted as problematic to the understanding of probabilistic relationships more generally (Bar-Hillel & Falk, 1982; Falk, 1992; Gillman, 1992; Margolis, 1987; Nickerson, 1996). The distinction between $p(D \mid H)$ and $p(H \mid D)$ appears to be one that is especially easily obscured by casual language use.

Consider the expressions “the probability of obtaining $D$ by chance” and “the probability that $D$ was obtained by chance.” It would hardly be surprising if these expressions were taken by most people to have the same referent. However, if what is meant by the first is “the probability that a process known to be a chance process, call it $H$, will produce $D$” and what is meant by the second is “the probability that a known event, $D$, was produced by a chance process, $H$,” they are quite different. The first is a reference to $p(D \mid H)$, and the second to $p(H \mid D)$.

It is also easy to find casual expressions of probabilistic relationships that lend themselves to more than one interpretation. “The probability that a chance process will produce $D$,” for example, can be taken to mean “given a chance process, the probability that it will produce $D$”; but it could also mean “the probability that a chance process will occur and will produce $D$.” Again, letting $H$ represent the chance process, the first expression can be represented as $p(D \mid H)$, and the second as $p(H \& D)$ or $p(H)p(D \mid H)$. Alternatively, consider the definition of power as “the probability of correctly rejecting the null hypothesis” (Harlow, 1997, p. 6). Power is the probability of rejecting the null hypothesis, given that it is false. “The probability of correctly rejecting the null hypothesis” might be interpreted to mean this, but it could also be taken to mean the absolute probability of correctly rejecting the null hypothesis, which is to say the joint probability of the null hypothesis being false and it being rejected. (In the context of computer simulation research, this ambiguity is sometimes avoided by referring to power as the proportion of false null hypotheses rejected [Lashley & Bond, 1997].)

I believe that much of the confusion about NHST and about what $p$ values mean derives from such ambiguities in casual language, some of which can be quite subtle. Although I have tried not to make statements in this article that are ambiguous or that reflect the beliefs that I am claiming are incorrect, I am far from confident that I have been successful in this regard.

Summary Regarding Misconceptions

The burden of the present article to this point has been to argue that there are many ways in which NHST can be, and is, misunderstood. Most, if not all, of the false beliefs mentioned here have been noted before, in some cases by many writers (e.g., Carver, 1978; Oakes, 1986; Schmidt, 1996). Investigators have documented misinterpretations in numerous published research articles (Dar, Serlin, & Omer, 1994) and in widely used texts as well (Cohen, 1994; Huberty, 1993).

Many of the conceptual difficulties that people have with NHST have their roots, I believe, in a failure to distinguish between absolute and conditional probabilities and, in particular, in failure to understand that the value of $p$ produced by conventional tests of statistical significance is a conditional probability—the probability of getting the obtained statistical result on the assumption that the null is true. A further source of confusion is failure to distinguish between the two conditional probabilities $p(D \mid H_0)$ and $p(H_0 \mid D)$, and treatment of the former as though it were the latter. Similar confusions pertain to $\alpha$, which is often treated as the absolute probability of a Type I error, or as the conditional probability of a Type I error given that the null has been rejected, when in fact it is the probability of a Type I error.
conditional on the null being true. Comparable points can be made regarding $\beta$.

If there is an additional key insight in understanding what NHST does and does not do, I think it is recognition that $p(D \mid H_0)$ tells us nothing about the value of $p(D \mid H_A)$ and, in particular, that $p(D \mid H_A)$ is not the complement of $p(D \mid H_0)$. If $p(H_0)$ and $p(H_A)$ are complements, then $p(H_A) = 1 - p(H_0)$ and $p(H_A \mid D) = 1 - p(H_0 \mid D)$. However, it is not necessarily the case that $p(D \mid H_A) = 1 - p(D \mid H_0)$. Theoretically, both $p(D \mid H_A)$ and $p(D \mid H_0)$ can vary from 0 to 1 independently. In particular, both $p(D \mid H_0)$ and $p(D \mid H_A)$ can be very small, and it is possible for $p(D \mid H_A)$ to be smaller than $p(D \mid H_0)$ when they are both small. The latter possibility is the basis of Lindley's paradox, described above.

I strongly suspect that many people believe that $p(D \mid H_0)$ and $p(D \mid H_A)$ are complements, or at least that if $p(D \mid H_0)$ is small, $p(D \mid H_A)$ must be large. If either of these relationships pertained, many of the beliefs that I have described as false would be true. Neither of these relationships necessarily pertains; in many cases of interest, it may be reasonable to assume that $p(D \mid H_A)$ is greater than $p(D \mid H_0)$ or even that $p(D \mid H_A)$ is much greater than $p(D \mid H_0)$, but this is an assumption; seldom is it possible to specify $p(D \mid H_A)$ precisely. Moreover, as Edwards (1965) has pointed out, typically in classical statistics the alternative to the null hypothesis is undefined, and attaching a probability conditional on an undefined hypothesis is seldom easy to do.

Lindley (1977) argued that when the probability of the data is small under both hypotheses under consideration, it makes sense to wonder whether perhaps some other hypothesis should be considered. DeGroot (1982) also noted that making an observation that is improbable under both hypotheses is likely to cause one to feel that there may be a good explanation of the observation other than those considered and to rethink one’s prior distribution of probabilities. On the other hand, in another context Lindley (1982) pointed out that “the comparison of small probabilities is the usual situation because most things that happen to us have low probability; we go through life experiencing rare events” (p. 335). In isolation, the fact that an event has low probability tells us very little about the nature of its cause.

To say that a result is statistically significant is to say that the probability of obtaining that result if there were no factors operating but chance is small. However, because the probability of obtaining that result in any case—by chance or otherwise—could be small, to say that a result is statistically significant is to say nothing conclusive about the probability that particular result was produced by chance. If one is willing to make the assumption that $p(D \mid H_A)$ is large relative to $p(D \mid H_0)$, then one has a legitimate basis for interpreting a small $p$ as evidence for increasing the likelihood of $H_A$ relative to that of $H_0$. Perhaps this assumption underlines many applications of NHST, but seldom does one see an explicit acknowledgment of it.

### Other Criticisms of NHST

Not all the criticisms that have been directed at NHST have focused on false beliefs about what it means, although many of them have. We turn now to some of the criticisms of other types that have been made.

#### A Priori Unlikelihood That the Null Hypothesis Is True

The reasonableness of NHST has been challenged by many writers on the grounds that the null hypothesis is very unlikely ever to be true and that statistically significant (though not necessarily large) differences (from 0 or any other hypothesized value) are almost assured on practically any dimension if one uses sufficiently large samples (Bakan, 1966; Berkson, 1938; Cohen, 1990; Grant, 1962; Hodges & Lehmann, 1954; Lindgren, 1976; Meehl, 1967, 1978; Murphy, 1990; Neyman & Pearson, 1928a, 1928b; Nunnally, 1960). The claim is that generally when an experiment yields data that do not permit the rejection of the null hypothesis at a prescribed level of statistical significance, it can be assumed that a significant difference would be obtained simply by increasing the sample size (Hays, 1994; Nunnally, 1960; Oakes, 1986). Thompson (1998) characterized the situation this way: “Statistical testing becomes a tautological search for enough participants to achieve statistical significance. If we fail to reject, it is only because we’ve been too lazy to drag in enough participants” (p. 799). Meehl (1990b, 1997) argued that the finding of significant correlations of nontrivial size between arbitrary variables with large data sets should not be surprising because everything really is related, to some degree, to everything else.

Not all psychologists agree with the claim that the null hypothesis is never true. Some have argued that demonstrating the tenability of the null hypothesis is
as legitimate a goal of research, though not necessarily as easily attained, as is demonstrating the tenability of any alternative hypothesis (Chow, 1996; Frick, 1995a). Frick (1995a), for example, argued that the null hypothesis can be true and should sometimes be accepted as such: "The null hypothesis is a valuable claim that psychology should want to accept, not merely fail to reject" (p. 132). Frick (1995a) conceded that there are instances in which the null hypothesis must be considered impossible but argued that this should not preclude its use in cases for which it is appropriate.

A context in which the possibility of obtaining evidence that the null hypothesis is true or nearly so is of considerable interest is medicine, as well as closely related areas (Malgady, 1998). Bartko (1991) gave several references in the biostatistics literature about "proving" the null hypothesis. Statistical approaches designed to establish the approximate equality (equality for practical purposes) of methods (as distinct from establishing the superiority of one over the other) that are based on classical NHST have been developed (Hauck & Anderson, 1986; Rogers et al., 1993; Westlake, 1988). These approaches are intended especially for use in the context of pharmaceutical research, where it is often important to establish that a new drug does not have undesirable side effects or to determine whether one drug or treatment is as clinically effective as another (despite, say, differences in cost, convenience, or other factors).

Hagen (1997) countered the claim that the null hypothesis is almost always false by contending that it is based on a misinterpretation of the null hypothesis.

The null hypothesis says nothing about samples being equal, nor does the alternative hypothesis say that they are different. Rather, when addressing group differences, the null hypothesis says that the observed samples, given their differences, were drawn from the same population, and the alternative hypothesis says that they were drawn from different populations. (p. 20)

Samples will always (or very nearly always) differ with respect to any measurable variable of interest, but this is true even of samples drawn from the same population; so the fact that they differ with respect to a specific measure in any particular instance is not evidence that they were drawn from different populations. Only if the magnitude of the difference is sufficiently great relative to the standard error of the difference to meet a conventional criterion will the conclusion that the samples are from different populations be drawn, and when samples really are drawn from the same population the likelihood that the null hypothesis will be rejected does not go to 1 as sample size increases. As Hagen said:

We have been taught that a sufficiently large N will detect differences no matter how tiny they may be. But what we may forget is that small differences will always be detected by a large N only under the alternative hypothesis, not under the null. When samples are drawn from the same population, the variance of absolute differences between or among such samples will become smaller as N becomes larger. This diminishing variance is reflected in a decrease in the variance of the particular test statistic from which we draw our sample statistic. Accordingly, Type-I error remains roughly constant no matter how large N becomes. (p. 20)

Another interpretation of the claim that the null hypothesis is almost always false is that any experimental manipulation—for example, differential treatment of two groups—is bound to have some effect, however small (Tukey, 1991). Hagen's counter to this claim is that although it may be that differential treatment will always have some effect, it may not have an effect on the dependent variable of interest, and it is only such an effect that will lead to rejection of the null hypothesis.

Another response to the claim that the null hypothesis is almost always false is that independent of the validity or invalidity of the claim, what is true of point null hypotheses need not be true of "small interval" hypotheses that can be approximated realistically by point nulls (Berger & Sellke, 1987; Hodges & Lehmann, 1954; Serlin & Lapsley, 1985; Wilson et al., 1967). More specifically, the argument is that even if the probability of a true point null were vanishingly small, something close to a null result—an almost-null result—would not be impossible a priori and for many purposes the null may serve as a useful proxy for an almost-null hypothesis:

Although we may specify a point null hypothesis for the purpose of our statistical test, we do recognize a more or less broad indifference zone about the null hypothesis consisting of values which are essentially equivalent to the null hypothesis for our present theory or practice. (Binder, 1963, p. 110)

Alternatively, as Meehl (1997) has said:

In practical contexts, when we have sufficient power (1 – β) so that there is not too big an asymmetry in the values of error rates α and β, we do want to make the "quasi-null" inference, not that H₀ as a precise point
value is literally true, but that something close to it is. (p. 395)

It has been argued that in most experiments in which the null hypothesis is tested the investigator is not really interested in the possibility of precisely zero effect but rather in whether whatever effect there might be is close enough to zero to be of no interest (Rindskopf, 1997). As to why, if this is the case, experimenters do not regularly test composite or range null hypotheses rather than point nulls, Rindskopf surmised that testing a point null is the much simpler process; although efforts to facilitate testing range nulls, which require assumptions about the distribution of a statistic when the null is false, have been made (e.g., Serlin & Lapsley, 1985, 1993). In the meantime, if sample size is not so large as to ensure detection of even negligibly small effects, testing of a point null hypothesis is likely to yield roughly the same result as would the testing of a small-range null.

Sensitivity of NHST to Sample Size

Although the likelihood that a true null hypothesis will be rejected does not increase with the sizes of the samples compared, the likelihood that a real difference of a given magnitude will result in rejection of the null hypothesis at a given level of confidence does. It is also the case that the smaller a real difference is, the larger the samples are likely to have to be to provide a basis for rejecting the null. In other words, whether or not one assumes that the null hypothesis is always or almost always false, when it is false the probability that a statistical significance test will lead to rejection increases with sample size.

This sensitivity to sample size has been the focus of some of the sharpest criticisms of NHST (Bakan, 1966; McNemar, 1960; Nunnally, 1960; Thompson, 1998). It means that conclusions drawn from experiments often depend on decisions experimenters have made regarding how many participants to run. Also, inasmuch as even very small real differences will be detected by sufficiently large samples, it is possible with very large samples to demonstrate statistical significance for differences that are too small to be of any theoretical or practical interest.

Because of the sensitivity of statistical significance to sample size, the practice of increasing the size of one's sample after performing an experiment that yielded a difference that failed to attain significance is generally considered poor form. As several writers have pointed out, if one is permitted to use an optional stopping rule, one can be quite certain of rejecting even a true null hypothesis if one goes on sampling for a sufficiently long time (I.J. Good, 1981/1983b; Greenwood, 1938; Robbins, 1952). The experimenter in this situation is somewhat like the gambler who is free to specify the size of the wager on every bet and to terminate the betting whenever he or she likes, thereby being effectively assured of winning.

The importance of being specific about the sample(s) one intends to use for experimental purposes is illustrated by the following situation, adapted from Berger and Berry (1988). Suppose an experimenter were to say, "I have just tossed a coin 17 times, obtaining 13 heads and 4 tails. Should I reject the null hypothesis (of an unbiased coin), and if so at what level of confidence?" One cannot answer this question without knowing the experimenter's original intent. If the intent were to toss the coin 17 times and make a statistical decision on the basis of the outcome, the answer provided by the standard approach to NHST is that the null hypothesis should be rejected at a confidence level of approximately .05. If, on the other hand, the experimenter had intended to toss the coin until 4 heads and 4 tails had been obtained, and the 4th tail happened to occur on the 17th toss, the null hypothesis should be rejected at a confidence level of .02.

The reason one gets two results from the sample is that although the sample is the same, it is drawn from two different populations. In the first case the population is all possible sets of 17 coin tosses; in the second the population is all possible sequences of tosses that are terminated on the first toss for which it is true that at least 4 of each possible outcome (heads or tails) have occurred. This population includes sequences ranging in number from 8 to infinity. Berger and Berry (1988) argued that standard statistical methods, which include NHST, "depend on the intentions of the investigator, including intentions about data that might have been obtained but were not" (p. 159).

Lindley (1993) made a similar point with respect to a modified form of Fisher's (1935) famous tea-tasting experiment. Suppose a tea taster, who claims to be able to tell by tasting a cup of tea with milk whether the tea or milk was put in the cup first, is tested six times and is right on the first five tries and wrong on the sixth: RRRRRW. The usual way of judging whether performance this good is likely on the basis of chance is to ask what the probability of getting five or more correct in six tries is; the answer, as conven-
tionally calculated, is .109, which assumes a sample space that includes all possible outcomes of 6 tests. However, suppose the experimenter’s intent was to continue the trials as long as the tea taster’s answers were correct—to terminate it after the first wrong response. In this case the sample space includes infinitely many sequences, and the probability of getting five or more correct responses before the first error is .031.

Lindley (1993) used this illustration of the ambiguity of the concept of outcomes that are more extreme than a specified outcome as a basis for arguing for the abandonment of the use of more extreme outcomes in statistical analyses—as when in NHST one considers the probability of obtaining a result equal to or more extreme than the one obtained. Inasmuch as he had already argued that considering only the probability of the exact outcome obtained is unsatisfactory because in some experiments the probability of every possible outcome is small, he then went on to argue that the better way to evaluate an outcome is the Bayesian approach of considering its probability conditional on the null hypothesis relative to its probability conditional on one or more alternative hypotheses—which is to say, likelihood ratios. Unlike the use of NHST, however, the use of the Bayesian approach requires that one or more alternatives to the null hypothesis and the probability of the data conditional on it (them) be specified. As Lindley (1993) said:

The Bayesian method is comparative. It compares the probabilities of the observed event on the null hypothesis and on the alternatives to it. In this respect it is quite different from Fisher’s approach which is absolute in the sense that it involves only a single consideration, the null hypothesis. (p. 25; see also Lindley, 1984)

Perhaps in part to preclude the use of strategies that permit an experimenter to decide on sample size on the basis of how an experiment in progress was turning out, some have argued that an investigator must specify experimental details—sample size, statistical tests, significance levels, interpretations of possible outcomes—in advance of collecting data. Fisher (1935/1956) expressed this idea as follows:

In considering the appropriateness of any proposed experimental design, it is always needful to forecast all possible results of the experiment, and to have decided without ambiguity what interpretation shall be placed upon each one of them. Further, we must know by what argument this interpretation is to be sustained. (p. 1512)

I. J. Good (1976/1983a) has challenged this position:

Many elementary textbooks recommend that test criteria should be chosen before observations are made. Unfortunately this could lead to a data analyst missing some unexpected and therefore probably important feature of the data. There is no existing substitute for examining the original observations with care. This is often more valuable than the application of formal significance tests. If it is easy and inexpensive to obtain new data, then there is little objection to the usual advice, since the original data can be used to formulate hypotheses to be tested on later samples. But often a further sample is expensive or virtually impossible to obtain. (p. 51) It's misleading to tell a student he must decide on his significance test in advance, although it is correct according to the Fisherian technique. (p. 54)

I. J. Good (1981/1983b) argued that “one cannot always sensibly determine a significance test in advance because, heretical though it may be in some quarters, sometimes the data overwhelmingly suggest a sensible theory after the data are examined” (p. 145).

There are techniques for analyzing data sequentially that are widely recognized to be legitimate that do not require the advance specification of sample size (Wald, 1945, 1947/1974). Bayesian approaches to belief revision are in this category. For the most part, such techniques are alternatives to NHST, but some work on application of sequential stopping rules to NHST has been done (Frick, 1998).

It can be argued that the question of the extent to which the details of a null hypothesis test should be specified in advance is beside the point. If, as some believe, we know before collecting data that the null hypothesis is false and that a sufficiently large sample would show it to be so in any particular case, what is the purpose of doing a test at all (Cohen, 1990; Neyman & Pearson, 1928a, 1928b; Nunnally, 1960)? If the null hypothesis is never true, then evidence that it should be rejected in any particular instance is neither surprising nor useful.

I. J. Good’s (1956/1983c) answer to the question of why one would want to do a significance test is, “That we wish to test whether the hypothesis is in some sense approximately true, or whether it is rejectable on the sort of size of sample that we intend to take” (p. 62). This point, he noted, is not usually made clear in textbooks on statistics, and in any event never formulated precisely. A similar answer has been given by Mulaik, Raju, and Harshman (1997):
It doesn’t matter if the null hypothesis is always judged false at some sample size, as long as we regard this as an empirical phenomenon. What matters is whether at the sample size we have we can distinguish observed deviations from our hypothesized values to be sufficiently large and improbable under a hypothesis of chance that we can treat them reasonably but provisionally as not due to chance error. (p. 80)

However, the more compelling response to the question of why to do a statistical significance test is denial of the claim that we know before doing an experiment that the outcome will be rejection of the null, if only the sample is sufficiently large. The argument, noted in the preceding section, that if the samples being compared are drawn from the same population with respect to the parameter of interest, the probability of getting a significant difference with respect to that parameter does not go to 1 as the sample size increases, seems to me conclusive.

**Faulty Logic**

Berkson (1942) criticized NHST on the grounds that it is illogical. Commenting on a textbook presentation of the null hypothesis procedure in which it was argued that the observation of a difference that would seldom occur by chance casts much doubt on the hypothesis of no difference, he described the case this way:

Consider [the argument] in syllogistic form. It says “If A is true, B will happen sometimes; therefore if B has been found to happen, A can be considered disproved.” There is no logical warrant for considering an event known to occur in a given hypothesis, even if infrequently, as disproving the hypothesis. (p. 326)

In fairness to the textbook writer, the claim was not that the observed result proved the null hypothesis to be false, but only that it cast much doubt on it. And in fairness to Berkson (1942), it needs to be said that his major complaint against NHST was that the logic of it “does not seem to accord with what would be the mode of reasoning in ordinary rational discourse, nor with the rationale of usual procedures as they are observed in the scientific laboratory” (p. 326). Berkson (1942) proposed the following principle as generally operative in scientific inquiry:

The finding of an event which is frequent under a hypothesis H0 can be taken as evidence in favor of H1. If H0 is a contradictory alternative to H1 for which the event would not be frequent, then per corollary the finding of the event is, in so far, evidence in disfavor of H0. (p. 327)

As applied to NHST, the principle means

If an event has occurred, the definitive question is not, “Is this an event which would be rare if H0 is true?” but “Is there an alternative hypothesis under which the event would be relatively frequent?” If there is no plausible alternative at all, the rarity is quite irrelevant to a decision, and if there is such an alternative, the decisive question is, “Would the event be relatively frequent?” (p. 327)

This is similar to the claim that $p(D \mid H_0)$ tells us nothing about $p(H_0 \mid D)$ in the absence of knowledge of or an assumption about $p(D \mid H_A)$. (Berkson, 1942, also developed an argument for being willing to accept—as distinct from failing to reject—the null hypothesis under specific conditions).

Cohen (1948) and Falk and Greenbaum (1995) contended more generally that the modus tollens form of argument—“If P then Q; not Q, therefore not P’”—which is valid with categorical premises, is invalid when the premises are probabilistic: “If P then probably Q; not Q; therefore probably not P” (“If $H_0$ is true then probably $p > .05$; $p < .05$; therefore probably $H_0$ is false”). Again, the point is that in order to say anything about the probability of $H_0$, given the occurrence of an event that has low probability if $H_0$ is true, we need to know, or to assume something about, the probability of the event if $H_0$ is false.

McDonald (1997) also considered statistical tests to be patterned after modus tollens: “If null hypothesis ($H_0$) then not-these-data (not-D), but D, therefore not-$H_0$ [which in the statistical context becomes] if $H_0$ then D improbable ($< \alpha$), therefore either not-$H_0$ or the improbable has occurred” (p. 200). McDonald agreed with others who have pointed out that this does not justify the conclusion that $H_0$ is improbable or unlikely, but he did allow that it may be rational (though possibly erroneous) to conclude not-$H_0$ rather than to conclude that the improbable has occurred; the choice here is between not-$H_0$ (the probability of which is unknown) and an event whose probability is known to be low.

Cortina and Dunlap (1997) acknowledged that modus tollens does not have the force with probabilistic premises that it has with categorical ones, but they argued that it can be used to good effect under certain conditions. They contended that it is approximately correct when the truth of the first premise’s antecedent is positively related to the truth of its consequent. The claim is that the more nearly correct it is to consider the antecedent of the conditional probabilistic
premise to be a cause or reason for the consequent, the more sense it makes to apply \textit{modus tollens}. If the consequent of the conditional premise is very likely to be true, independent of the truth or falsity of the antecedent, the form has little force. "If \( X \), then that person is probably not a member of Congress" seems likely to be true, almost independently of what is substituted for \( X \), so \( X \) cannot be considered a reason for expecting the consequent to be true. This stands in contrast to the statement "If the experimental manipulation did not work, then \( p \) would probably be greater than \( .05 \)," which seems to be offering the experimental manipulation not working as a reason for getting a value of \( p \) greater than \( .05 \). Cortina and Dunlap argued that the application of \textit{modus tollens} is justified in the latter case but not in the former; they suggested that the latter case is the more representative than the former of the conditions under which NHST is usually done and concluded that "the typical approach to hypothesis testing does not violate the relevant rule of syllogistic reasoning to any great degree" (p. 166).

Hagen's (1997, 1998) position with respect to the argument that NHST lacks logical validity is that "arguments can be reasonable and defensible even when they are not logically valid in a formal sense" (1997, p. 22), I. J. Good (1982) also considered classical NHST to be useful, despite being logically flawed: "Logically there is something wrong with the use of tail-area probabilities, but I still find them useful because of the well-known difficulties about priors in the Bayesian position" (p. 342).

\textbf{Noninformativeness of Test Outcomes}

Another objection to NHST is that such tests provide relatively little information about the relationship between the dependent and independent variables. They do not, as we have seen, provide a measure of the size of an effect, nor do they reveal the strength of the relationship between dependent and independent variables. They give evidence only of whether a statistically significant effect has been obtained and, if so, of the direction of the effect. At least when the null hypothesis is the hypothesis of zero difference (zero effect, zero correlation), statistical significance provides at best evidence against the hypothesis of no difference (effect, correlation), which is very little information indeed. As Abelson (1997b) has said, "Typically, mere difference from zero is totally uninteresting" (p. 121).

In contrast, a regression analysis gives an indication of the degree of relatedness of variables (Cohen, 1977; Cohen & Cohen, 1983). The ratio of between-treatments variance to total variance—the proportion of total variance in a dependent variable that can be attributed to treatments—has also been suggested as an indication of strength of relationship between independent and dependent variables (R. Good & Fletcher, 1981; Hays, 1994; Stocks, 1987). A Bayesian analysis can provide a posterior probability for each of a set of hypotheses of interest (Bakan, 1966; Cronbach & Snow, 1977; but only, of course, if the values of the variables required by the computation are known or assumed).

\textbf{Inappropriateness of All-or-None Decisions Regarding Significance}

Many writers object to the sharpness of the distinction that is made between significant and nonsignificant results (Eysenck, 1960; Frick, 1996; Glass, McGaw, & Smith, 1981; Grant, 1962; Nunnally, 1960; Rossi, 1997; C. D. Wickens, 1998). According to some interpretations of the conventional rules of application of NHST, a result that yields a \( p \) value only slightly greater than the \( \alpha \) level is to be given the same treatment as one that is much greater. Many researchers find it very difficult to follow this principle, however, and insist on distinguishing between "marginally significant" and "nonsignificant" results. It does seem a little strange to consider a difference with a \( p \) of \( .05 \) to represent something real while dismissing one with a \( p \) of \( .06 \) as due to chance. Nevertheless, several inquiries into how psychologists interpret the results of statistical significance tests have shown a "cliff characteristic" at \( .05 \), according to which reported confidence in a finding drops abruptly when \( p \) becomes larger than this value (Beauchamp & May, 1964; Rosenthal & Gaito, 1963, 1964); cliff characteristics of lesser magnitude have also been found for \( p \) values of \( .01 \) and \( .10 \) (Minturn, Lansky, & Dember, 1972; Nelson, Rosenthal, & Rosnow, 1986).

Rozeboom (1960) objected to NHST on the grounds that it treats acceptance or rejection of a hypothesis as though this were a decision one makes on the basis of the experimental data; the experimenter's task, he argued, is not that of making a binary decision either to accept or to reject a tested hypothesis, but rather that of determining how the experimental outcome changes the probability that the hypothesis is true. The scientist "is fundamentally and inescapably committed to an explicit concern with the problem of inverse probability" (p. 422). As a matter of fact, Rozeboom (1960) contended, researchers do not apply
the principles of NHST in forming and refining their own beliefs about the tenability of hypotheses. I. J. Good (1981/1983b) made a similar point in contending that it is not always sensible either to accept or to reject a hypothesis in a sharp sense. Rosnow and Rosenthal (1989b) maintained that dichotomous significance testing has no ontological basis and that the strength of evidence for or against the null hypothesis must be considered a fairly continuous function of the magnitude of $p$.

The dichotomization of experimental results into those that prove to be statistically significant and those that do not, and the attendant strong bias for publishing only the former, are seen by some critics of NHST as very detrimental to the advance of psychology as a science. Essentially ignoring null findings, they argue, inhibits the accumulation of knowledge across studies (Meehl, 1978; Rossi, 1997).

**Arbitrariness of the Decision Criterion**

Closely associated with the objection regarding the sharp distinction between significance and nonsignificance is concern about the arbitrariness of the $\alpha$ criterion (Glass et al., 1981; Rozeboom, 1960). The $\alpha$ criterion that is most widely recommended is .05 (Cowles & Davis, 1982). The grip that this number has had on the research community for decades has been parodied by Rosnow and Rosenthal (1989b):

> It may not be an exaggeration to say that for many Ph.D. students, for whom the .05 alpha has acquired almost an ontological mystique, it can mean joy, a doctoral degree, and a tenure-track position at a major university if their dissertation $p$ is less than .05. However, if the $p$ is greater than .05, it can mean ruin, despair, and their advisor’s thinking of a new control condition that should be run. (p. 1277)

My experience suggests that .05 is treated by many researchers today as an upper bound on what should be considered statistically significant, but relatively few specify it as $\alpha$ in advance of collecting (or reporting) data and then report all results relative to that criterion. More commonly, researchers report a variety of $p$ values in the same study, although they typically refer only to those that are less than .05 as statistically significant.

The guidance provided to authors by the *Publication Manual of the American Psychological Association* (4th ed.; American Psychological Association [APA], 1994) allows considerable latitude in the selection (or not) of an $\alpha$ level and the reporting of $p$ values. Whether this is a good idea has been a topic of debate (Labovitz, 1968). Some writers have urged that, at least when the results of statistical significance tests are to be used as a basis for decisions that matter, the costs and benefits associated with the various ways of being right and wrong should be considered in deciding what the null hypothesis should be and in setting the $\alpha$ level (Cox, 1958; Neyman, 1942; Oakes, 1986; Skipper, Guenther, & Nass, 1967).

In contrast, others believe that freedom to select one’s own $\alpha$ level adds an undesirable element of subjectivity to the process of hypothesis evaluation (Frick, 1996) and permits different investigators to draw conflicting conclusions from the same data (Cox, 1977). Rozeboom (1960) captured this concern this way: “Surely the degree to which a datum corroborates or impugns a proposition should be independent of the datum-assessor’s personal temerity” (p. 420). The establishment of a widely adhered-to criterion, say, an $\alpha$ of .05, is seen by some as an attempt at standardization in the interest of objectivity, as “an admittedly arbitrary attempt to standardize a bias against alternative hypotheses . . . a deliberate attempt to offer a standardized, public method for objectifying an individual scientist’s willingness to make an inference” (W. Wilson et al., 1967, p. 191). W. Wilson et al. contended that this approach is more objective and less subject to variability deriving from individual differences in belief states among experimenters than a Bayesian approach that requires the assignment of personal probabilities would be. Chow (1998a) defended the use of a strict $\alpha$ criterion by arguing that it is analogous in importance to the maintenance by a teacher of a passing grade.

**Test Bias**

The question of bias in NHST is an interesting one, in part because the convention of selecting a small $\alpha$ is generally viewed as reflective of a strong bias against rejection of a true null hypothesis. It has been argued, however, that classical NHST (in contrast to Bayesian tests) is in fact strongly biased against acceptance (or nonrejection) of the null hypothesis (Edwards, 1965; Edwards et al., 1963; Lindley, 1993): “Classical procedures quite typically are, from a Bayesian point of view, far too ready to reject null hypotheses” (Edwards et al., 1963, p. 225).

The bias against acceptance of the null hypothesis of which Edwards and his colleagues (Edwards, 1965; Edwards et al., 1963) spoke has been noted in this article in the context of the discussion of the false
belief that $p$ is the probability that the null hypothesis is true. As was pointed out there, given $p(H_0) = .5$ and certain assumptions about how $D$ is distributed, $p$ is invariably smaller than $p(H_0 | D)$ and in extreme cases can differ from it by a large amount. A consequence of this difference between $p$ and $p(H_0 | D)$, from a Bayesian point of view, is that when $H_0$ is rejected at some significance level, say, $x$, $p(H_0 | D)$ may be much larger than $x$. As also noted, it is even possible for $H_0$ to be rejected at an arbitrarily small value of $p$ under conditions in which a Bayesian analysis would show $p(H_0 | D)$ to be close to 1 (Lindley's paradox), although the conditions under which this happens do not seem likely to often characterize psychological experiments.

Many opponents of NHST have argued against it on the grounds that it biases the reporting of results of experimentation: Because of the general practice of not publishing results that did not attain statistical significance at at least the .05 level, many real effects are not reported, according to this argument. (More on this point later.) As W. Wilson et al. (1967) have pointed out, bias is a relative term, and the same test can be biased in more than one way. In particular, as compared with a Bayesian analysis, a classical analysis can be biased against the null, whereas as compared with a sensitive experiment an insensitive experiment can be biased for the null.

Possible Inflation of Type I Errors in the Literature

Nearly 90% of the respondents (all active social-psychological researchers) to a survey conducted by Greenwald (1975) reported being less likely to submit for publication failures to reject the null hypothesis than successes in doing so. This reluctance to report null results could be due at least in part to the assumption—undoubtedly valid—that editors are generally not enthusiastic about publishing such results. It may also be due in part to a tendency of researchers to interpret failure to reject the null hypothesis as uninformative and perhaps the result of flawed methods (Greenwald et al., 1996).

With an $\alpha$ of .05 we expect about 5% of those instances in which the null hypothesis is true to yield a "statistically significant" effect, that is, in reality, a Type I error. If statistically significant effects (rejections of the null) are much more likely to be published than failures to attain statistical significance (failures to reject the null), this means that when the null is true only those analyses that have produced Type I errors are likely to be published. It has been proposed that this can lead to "$\alpha$ inflation," which is to say that as that are reported in the literature can be spuriously small and can underestimate the probability of reporting chance effects as real, because they do not take the incidence of unpublished null results into account.

To understand and evaluate this concern, we need to distinguish eight conditions defined by the combinations of (a) the null hypothesis actually being true or false, (b) it being judged to be true or false, and (c) whether the outcome of the test was published. The situation may be represented as in Table 4. By definition Type I error is rejecting the null hypothesis when it is true, represented by cells B and D in the table. Theoretically the probability of a Type I error conditional on the null being true is the ratio of the total number of times a true null is rejected to the total number of times the null is true, which, letting the letters in the cells of Table 4 represent the numbers of events in those cells, is $(B+D)/(B+D+F+H)$. If we knew the number (or percentage) of cases in each of these cells, we would expect this ratio to be close to the indicated value of $\alpha$ (in this case .05), if the assumptions of the statistical test that was used were always met. We do not know the numbers in these cells, but concern about the "file-drawer" problem, as Rosenthal (1979) has called it, rests on the assumption that tests that yield statistical significance are more likely to be represented by the first row of the table than by the second and that those that fail to yield significance are more likely to be represented by the

<table>
<thead>
<tr>
<th>Publication</th>
<th>Truth state of $H_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$ rejected ($p &lt; .05$) and published</td>
<td>True</td>
</tr>
<tr>
<td>$H_0$ rejected ($p &lt; .05$) and not published</td>
<td>False</td>
</tr>
<tr>
<td>$H_0$ not rejected ($p &gt; .05$) and published</td>
<td>False</td>
</tr>
<tr>
<td>$H_0$ not rejected ($p &gt; .05$) and not published</td>
<td>True</td>
</tr>
</tbody>
</table>

Table 4

Eight Combinations of Truth States of $H_0$ and Reporting Possibilities
fourth row than by the third, and more specifically that $H$ is large relative to both $D$ and $F$. If the latter assumption is valid, then $B/(B+F) > (B+D)/(B+D+F+H)$; which is to say that if both of these ratios were known the first, which reflects the published literature, would overstate the actual probability of Type I error conditional on the null hypothesis being true, relative to what is likely to be the case when all published and unpublished tests are taken into account.

Of course, none of the numbers (or percentages) of all null hypothesis test outcomes that fall within any of these cells is known. Conceivably we could know the sum of $A$ and $B$ and the sum of $E$ and $F$. However, if we knew the sum of $A$ and $B$, we would not know how to partition it between $A$ and $B$, and the same holds for $E$ and $F$. It is not the case that $A$ and $B$ could be inferred from a knowledge of $A+B$ and application of the relationship $\alpha = B/(A+B)$ because, even in theory, $\alpha$ is not intended to represent the relative frequency of Type I error in published work only.

In fact, it is not possible to determine the frequency with which Type I errors are made relative to the frequency with which the null hypothesis is true—the probability of Type I error conditional on the null being true—but if one is willing to assume that most of the experiments that end up in file drawers are represented by the last row of the table ($G$ and $H$), one can develop a plausible scenario for how this might generate concern for the effect of the file-drawer problem on beliefs about the incidence of Type I error and how it relates to $\alpha$.

Imagine, for the sake of an extreme illustration, that 100 experiments were conducted and that in each case the null hypothesis was true. About 5 of these would be expected to yield differences significant at the .05 level. If only the experiments yielding significant differences were published and the other 95 were not, the likelihood that the differences reported as significant were actually Type I error would be grossly underestimated by $p$.

Despite the foregoing, I do not believe $\alpha$ inflation to be a problem, for the following reasons. First, as already noted, $p$ is not an estimate of the probability that if one has rejected the null hypothesis one has made a Type I error, nor is $\alpha$ the probability of making a Type I error in a particular experiment or across a set of experiments; $p$ is the probability of obtaining a specified result if the null hypothesis is true. Type I error is rejection of the null hypothesis if it is true, and $\alpha$ is the risk that one is willing to take of making a Type I error when the null hypothesis is true.

Second, there is good reason to believe that in general, worry about $\alpha$ inflation notwithstanding, the probability of Type I error is considerably less than $\alpha$. I believe the following assumptions would be generally accepted by most parties in the debate about NHST, whatever their position regarding its merits: (a) For a large majority of psychological experiments, the null hypothesis is false; (b) considering all experiments done, published and unpublished, Type I error is more common than Type II error; and (c) experiments that yield statistically significant results are much more likely to be published than experiments that do not.

The implications of these assumptions are illustrated in Table 5. For simplicity, suppose that an experiment is published if and only if it yields a result that is statistically significant at the .05 level. For purposes of the illustration, it is assumed that the null hypothesis is five times as likely to be false as to be true and that the probability that the null will be rejected at the .05 level if it is false is .5. Given these assumptions (plus the probability of .05 of a Type I error, given that the null is true), the probability that a statistically significant result is a Type I error, given that it is published, is 5/255, or about .02. Readers who find the assumptions of the illustration implausible may wish to substitute others more in keeping with their intuitions. I find it difficult to imagine a plausible set of values that would result in the relative frequency of Type I errors in published experiments.

<table>
<thead>
<tr>
<th>Decision regarding $H_0$</th>
<th>Truth state of $H_0$</th>
<th>False Frequency</th>
<th>True Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rejected ($p &lt; .05$) and published</td>
<td>Correct rejection</td>
<td>250</td>
<td>Type I error</td>
<td>5</td>
</tr>
<tr>
<td>Not rejected ($p &gt; .05$) and not published</td>
<td>Type II error</td>
<td>250</td>
<td>Correct nonrejection</td>
<td>95</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>500</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
being much larger than \( \alpha \). Of course, if one believes that the null hypothesis is always false then \( \alpha \) inflation is not a worry, because Type I errors cannot occur. However, those who believe the null hypothesis is always false do not favor NHST in any case.

**Presumed Frequency of Type II Error**

As already noted, Type II error is defined as failure to reject a null hypothesis that is false. The convention of adopting a strict criterion for rejecting the null hypothesis, say, an \( \alpha \) of \( .05 \), especially when coupled with statistical significance testing with low power because of small sample sizes, is assumed to mean that this type of error occurs frequently (Clark-Carter, 1997; Hunter, 1997; Rosnow & Rosenthal, 1989b; Sedlmeier & Gigerenzer, 1989). This can be especially problematic in situations in which a Type II error is likely to be as costly as, or even more costly than, a Type I error, and this may often be the case in applied settings (C. D. Wickens, 1998).

Power is difficult if not impossible to determine exactly in many cases, but several investigators have estimated that it typically is low—in the .4 to .6 range (Cohen, 1965, 1988; Schmidt, Hunter, & Urry, 1976)—and that this has been the case since it was first pointed out by Cohen (1962; Rossi, 1990; Sedlmeier & Gigerenzer, 1989). Belief that power is typically low and that the null hypothesis is nearly always false leads to the conclusion that the absolute probability of Type II error could be as high as .50 (Cohen, 1962; Schmidt, 1992).

This reasoning has been criticized on the grounds that it fails to distinguish between power to detect effects of hypothetical magnitude and power to detect actual effects and that it is based on the former, which could be quite different from the latter (Mulik et al., 1997). By definition a Type II error is committed only when an actual effect does not yield a statistically significant outcome; the fact that a test had low power does not mean that a real effect has gone undetected if one recognizes the possibility that there was no effect to detect (Erwin, 1998). Moreover, estimates of average power are based on hypothetical effect sizes that are assumed to be equal; Abelson (1997a) pointed out not only that true effect sizes vary, but also that the hypothesis that they do not is itself a null hypothesis, and if—as some critics of NHST claim—all null hypotheses are false, it is therefore self-defeating. Such considerations speak to the tentativeness of estimates of the frequency of Type II errors, but they do not rule out the possibility that it is substantial.

Because of the bias against publishing results yielding \( p \) values greater than \( .05 \) (Atkinson, Furlong, & Wampold, 1982; Greenwald, 1975), results that are actually Type II errors, which conceivably could include potentially important and interesting findings, are often buried along with true null effects without ever being published. One proposal for addressing this problem has been to publish the results of all experiments, whether they attain statistical significance or not, and rely on meta-analyses to draw conclusions on the basis of large bodies of results in the aggregate (Schmidt & Hunter, 1997).

Making statistical significance a requirement for publication may create a biased literature in at least three ways: (a) Because failures to replicate (failures to get statistically significant effects in attempts to replicate previous results) are seldom published, Type I errors are likely to go undetected (H. H. Clark, 1976; Greenwald, 1975); (b) because the criterion is generally set fairly high (at least .05), many real effects are not reported, which is to say the Type II error is frequently made; (c) because, other things equal (sample sizes and within-sample variance), large differences are more likely than small ones to yield \( p < .05 \), the differences that are reported in the literature may be larger on average than the population effects they ostensibly represent.

One concern relating to the last problem is that, with rare exceptions, only studies that have obtained statistically significant results are represented in review articles intended to summarize findings in specific areas of research. Such "nose counting" (Meehl, 1978) has been criticized for two different reasons: (a) It overlooks many experiments that failed to attain statistically significant results (possibly because of small sample sizes) and therefore may underestimate the strength of the aggregate evidence that exists for a finding (Cooper & Rosenthal, 1980; Hedges & Olkin, 1980); and (b) inasmuch as it attends only to effects that were large enough to attain statistical significance even sometimes with small samples, it can overestimate the size of a population effect (Abelson, 1997a; R. J. Harris, 1997a; Lane & Dunlap, 1978; Schmidt, 1992, 1996). Ironically both aspects of the problem can be exacerbated by the use of statistical procedures designed to correct for multiple applications of a test statistic (Cohen, 1994). However, this may not be a worry, because although the need to make such corrections has been argued for a long time (B. Wilkinson, 1951) and several techniques are available for doing so (Dunn, 1961; Hayter, 1986; Holm, 1979;
null hypothesis significance testing

Shaffer, 1979), it is often—perhaps more often than not—ignored in the reporting of statistical significance levels.

R. J. Harris (1997a) pointed out that overestimation of effect sizes in the literature could conceivably influence researchers to design replication experiments with too little power. If they believe an effect to be larger than it actually is and design an experiment that has sufficient power to detect an effect of the believed size, it may not have enough power to detect an effect of the actual size. An example of harm that can be done by disregarding the problem of Type II error has been given by Freiman, Chalmers, Smith, and Kuebler (1978), who demonstrated the ease with which therapeutically effective interventions can be missed in clinical trials with too few participants. Recognition of the often overlooked cost of Type II errors would seem to dictate the use of criteria that are not so extremely protective against errors of Type I. More generally, decision criteria should be set so as to take into account the consequences of the various possible ways of being wrong and with recognition that the relative costs of Type I and Type II errors can vary considerably from situation to situation.

Ease of Violating Assumptions of Statistical Tests

Statistical tests, at least parametric statistical tests, invariably rest on certain assumptions. Student's t test and the analysis of variance (ANOVA) involve assumptions regarding how the variables of interest are distributed. Student's t, for example, tests the hypothesis that two samples of measures of interest were randomly drawn from the same normally distributed population with a specific variance. Generally, however, neither the shape of the population of interest nor its variance is known; its distribution is assumed to be normal and its variance is estimated from the variances of the samples. For application of the test to be legitimate, the samples should be normally distributed and they should have roughly equal variances. Procedures have been developed for determining whether samples meet the requirements and for transforming the raw data in certain ways when they do not meet them so they will. The degree to which researchers ensure that data satisfy all the assumptions underlying the significance tests they apply to them is not known; my guess is that it is not high.

Random sampling from a population and random assignment to experimental treatment groups are important assumptions underlying ANOVA and similar statistical significance tests, and violation of one or both of these assumptions can invalidate the results of such tests (Winer, Brown, & Michels, 1991). This is a point that gets little, if any, attention in many statistical texts (Shaver, 1993). Sampling issues have been discussed by Lindquist (1940), Selvin (1957), and Edgington (1966, 1995). The problem of failing to satisfy assumptions underlying statistical tests is a complicated one because tests differ considerably in their sensitivity to violations of assumptions, and the extent to which specific tests are affected by violations of specific types is sometimes a subject of debate (Thompson, 1993).

Influence on Experimental Design

Carver (1978) argued that the preeminence of NHST as the method of data analysis in psychology encourages the design of experiments that lend themselves to this type of analysis. Some believe that reliance on NHST promotes the development of weak theories that are incapable of supporting bold and precise predictions (Dar, 1987, 1998). Gigerenzer (1998b) made the point this way:

The single most important problem with null hypothesis testing is that it provides researchers with no incentive to develop precise hypotheses. To perform a significance test, one need not specify the predictions of either one's own research hypothesis or those of alternative hypotheses. All one has to do is test an unspecified hypothesis (Hₐ) against "chance" (H₀). In my experience, the routine of testing against chance using NHSTP [the null hypothesis test procedure] promotes imprecise hypotheses... Testing an unspecified hypothesis against chance may be all we can do in situations where we know very little. But when used as a general ritual, this method ironically ensures that we continue to know very little. (p. 200)

Yates (1951) has suggested that the emphasis that has been put on statistical significance has promoted concentration on problems that have little or no practical significance. The types of questions that drive research are always determined, to some extent, by methodological considerations as well as by the theoretical or practical importance that researchers attach to them. There is little point, for scientific purposes, in asking questions that are beyond available methodologies to answer. However, if NHST is as flawed as its most severe critics suggest, letting its availability and use dictate the design of experiments seems unfortunate at best.
Defenses of NHST and Recommendations
Regarding Its Use

The abundance of criticisms notwithstanding, NHST is not without its defenders (Baril & Cannon, 1995; Chow, 1987, 1988, 1989, 1991, 1996, 1998a, 1998b; Cortina & Dunlap, 1997; Cux, 1977; Dixon, 1998; Frick, 1996; Giere, 1972; R. J. Harris, 1997a, 1997b; Kalbfleisch & Sprott, 1976; Mulaik et al., 1997; Robinson & Levin, 1997; Sohn, 1998b; W. Wilson et al., 1967; Winch & Campbell, 1969). Proponents of NHST generally acknowledge that it has limitations, and few if any argue that it should be the only analytic tool in the researcher’s repertoire. Some point out that many of the criticisms of NHST are not so much criticisms of NHST per se but criticisms of some of its users and misuses. It is not the fault of the process, they contend, if some of its users misunderstand it, expect more from it than it promises to deliver, or apply it inappropriately.

Abelson (1995, 1997a, 1997b) has defended NHST when used judiciously, not as an unequivocal determinant of what is worth reporting but as a means of helping to justify claims that specific effects were unlikely to have been obtained by chance. He argued that the noteworthiness of an experimental finding is determined by a variety of criteria, including the magnitude, generality, and interestingness of the effect. All statistics, in his view, should be treated as aids to principled argument. He saw NHST as not the only, but an essential, tool in the researcher’s kit: “Significance tests fill an important need in answering some key questions, and if they did not exist they would have to be invented” (1997b, p. 117). Dixon (1998) contended that despite justified claims of the limitations of NHST and misinterpretations of test results, p values can convey useful information regarding the strength of evidence: “Although p values are not the most direct index of this information, they provide a reasonable surrogate within the constraints posed by the mechanics of traditional hypothesis testing” (p. 391).

Macdonald (1997) described both the Fisherian and Neyman–Pearson approaches to statistical inference as being “concerned with establishing that an observed effect could not plausibly be accounted for by sampling error” (p. 334). When an achieved significance level—the probability of obtaining an effect as extreme as the one observed—is sufficiently low, the hypothesis that the obtained effect could plausibly be due to sampling error is rejected, and rejection of this hypothesis entails also rejection of the hypothesis of a true effect opposite in direction to that observed. Macdonald defended the Fisherian approach but contended that acquired significance levels should be seen as a guide to whether an effect has been demonstrated and not taken as the sole criterion.

Frick (1996) suggested that NHST is ideal for supporting ordinal claims but not for evaluating models that make quantitative predictions. By an ordinal claim, he meant a claim that does not specify the size of an effect but that specifies “only the order of conditions, the order of effects, or the direction of a correlation” (p. 380). Tukey (1991) contended that it is really the direction of an effect rather than the existence of one that the t test helps decide. Others have argued that if the question of interest is whether a difference is positive or negative, NHST is a suitable approach to finding out (Abelson, 1997b; R. J. Harris, 1997a, 1997b; W. Wilson et al., 1967).

Chow (1987, 1988, 1989, 1991, 1996) has defended NHST as a means of evaluating hypotheses, which he noted are usually expressed in the form of an ordinal relation such as a statement that performance is better (or worse) under an experimental condition than under a control condition. He made a distinction, however, between the problem of testing a hypothesis and that of evaluating a theory. In his view, statistical significance is relevant to hypothesis testing, but theory evaluation requires integration of the results of tests of many hypotheses that are deductible from the theory in question and of other relevant information as well. Hypothesis testing appropriately ends in a binary decision as to whether the data in hand are consistent with the hypothesis or not; a theory is corroborated incrementally by the accumulation of evidence from converging operations designed to test a variety of hypotheses that are deductible from it. Chow (1989) distinguished too, between the support a theory receives from a particular experiment and the theory’s overall status, the latter being determined “by how often it has withstood attempts to falsify it, how well it fares when compared to contending theories, how extensive the converging operations in its support are” (p. 164).

In his defense of NHST, Chow (1996, 1998a, 1998b) distinguished hypotheses at several levels of abstraction—from lowest to highest: statistical, experimental, research, and substantive—and argued that some of the criticisms that have been leveled at NHST are based on failure to recognize the difference between statistical hypotheses and higher level hypotheses. Other criticisms of NHST, he contended,
are not appropriately aimed at NHST per se but at inferential processes that are outside the domain of statistics. It is especially important to recognize, Chow (1998b) argued, that testing a statistical hypothesis is not tantamount to testing a substantive (explanatory, theoretical) hypothesis. Corroboration (refutation) of a substantive hypothesis involves showing that evidence (e.g., experimental data) is consistent (inconsistent) with that hypothesis; showing that a statistical hypothesis is or is not tenable can be a factor in that process, but only one among many. The limited but essential role of NHST, in Chow’s view, is that of ruling out chance (at a specified level of strictness) as an explanation of data:

A significant result indicates a discernible effect in the sense that chance influences are excluded with reference to a well-defined criterion. By the same token a significant effect is a genuine effect in the sense that it is not brought about by chance factors. NHST is the indispensable tool for assessing whether or not chance influences can be excluded as an explanation of the data. (1998b, pp. 327–329)

The claim that the role of NHST is to rule out chance as an explanation of data is easily interpreted as supportive of the belief that p represents the probability that chance is the operative factor given the data, \( p(H_0 \mid D) \); and we have already seen that this belief is false, although p can be a reasonable proxy for \( p(H_0 \mid D) \) given certain assumptions that appear to often be plausible. Furthermore, psychologists use NHST to test substantive hypotheses in theory-corroborating studies whether or not it is adequate to that task. Dar (1998) spoke to the latter point this way: “The move from the substance to the statistical hypothesis is a swift one: researchers interpret statistical significance as confirming the substantive hypothesis and therefore as corroborating the theory from which the hypothesis was derived” (p. 196). This, of course, is a criticism of the behavior of researchers and not of NHST per se.

In an attempt to account for the popularity of NHST, Greenwald et al. (1996) argued that it provides a dichotomous outcome that can be used for decision making, that p is a common-language translation for a variety of statistics (having one measure of how surprising a result should be under the hypothesis of no effect makes it unnecessary to deal with many such indicants), and that p also constitutes a measure of confidence in the replicability of null hypothesis rejection. (As noted in the foregoing, the question of whether p is a justified measure of confidence in replicability is a contested one.) These are all desirable properties in the eyes of NHST users.

Mulaik et al. (1997) argued that much of the opposition to NHST stems from confusion on a variety of subjects. They noted several beliefs, including some of those mentioned in the first part of this article, all of which they considered to be incorrect. They contended that many of the criticisms of NHST really pertain to misconstruals or misuses of significance tests and thus may speak to the incompetence of the misusers, but do not demonstrate the invalidity of the approach per se. Criticisms of “abusive misconceptions of the use and interpretation of significance testing,” Mulaik et al. insisted, “can hardly be regarded as criticisms of significance testing properly understood and applied” (p. 74). The fact that there are arsonists in the world does not make fire a bad thing. A similar position has been taken by Rossi (1997).

A common misuse of significance tests is to conclude that a difference due to two treatments, A and B, is statistically significant on the grounds that the effect of A is significantly different from zero and that of B is not. Regarding whether this type of confusion should be held against the test per se, Abelson (1997b) had this comment: “If a legal case were being brought against the significance test, the charge here would be that the test is an ‘attractive nuisance,’ like a neighbor’s pond in which children drown. It tempts you into making inappropriate statements” (p. 120).

As for the contention that NHST should be banned, Mulaik et al. (1997) took the opposite position:

We cannot get rid of significance tests because they provide us with the criteria by which provisionally to distinguish results due to chance variation from results that represent systematic effects in data available to us. As long as we have a conception of how variation in results may be due to chance and regard it as applicable to our experience, we will have a need for significance tests in some form or another. (p. 81)

Calls for the banning of NHST, Mulaik et al. argued, stem from a conception of NHST that overlooks the provisional nature of conclusions that result from it.

In addition to defending the use of NHST, proponents of it have offered several suggestions regarding how it might be used more effectively. I turn next to a consideration of some of these suggestions.

Make Greater Use of Non-Nil Null Hypotheses

Some of the problems associated with NHST disappear or at least diminish if the null hypothesis is
defined as something other than a hypothesis of zero difference, zero effect, or zero correlation (the nil null hypothesis). Ideally one would like to be able to make precise quantitative predictions of the effects of experimental manipulations, but most areas of psychology do not permit a high degree of precision at their present level of development. It does not follow that one must settle for the nil null hypothesis in all cases, however. Quasi-quantitative predictions, of rough magnitudes of effects, could help advance the field (Blaich, 1998).

Mulaik et al. (1997) distinguished between the problem of deciding what hypothesis one should test and that of applying NHST procedures appropriately to whatever that hypothesis is. The same writers suggested that one way to facilitate knowledge accumulation in psychology is to use what is learned in experiments to establish values for non-nil null hypotheses in subsequent experiments: "A hypothesis one ought to test is that the effect is equal to the value estimated in the previous study, which one judged to be significantly different from a zero effect" (p. 98). This possibility has also been pointed out by Greenwald et al. (1996). Mulaik et al. noted that with existing ANOVA methods and programs it is not easy to specify hypotheses in terms of previously found effects, and they suggested the desirability of some modernization of ANOVA in this regard. Some will see this suggestion as an argument for applying a Bayesian approach to data analysis.

**Use Range Null Hypotheses**

An alternative to the use of the point null hypothesis that has been urged by several writers is the use of a range null hypothesis, which involves designating a range of values that will be considered effectively null (Hodges & Lehmann, 1954; Meehl, 1967; Serlin, 1993; Serlin & Lapsley, 1985, 1993; Yeaton & Sechrest, 1986). Serlin (1993) argued that how wide the range should be depends on the state of the theory of interest but that, in any case, it should be specified in advance of an experiment. Some writers have taken the position that when testing a point null hypothesis researchers typically mean to treat it as a proxy for a range hypothesis anyway, which is to say that they usually do not seriously consider a difference of exactly zero to be probable, but they use zero to represent anything close (without specifying how close) to it. Cortina and Dunlap (1997) surmised that although it would be better if researchers specified the range of values they would consider within the "good-enough belt" described by Serlin and Lapsley (1985), "the vast majority of research that has focused on the nil would have been very much the same even if an alternative null value had been used" (p. 167).

**Be Specific About the Alternative(s) to the Null Hypothesis**

The predominant NHST paradigm leaves the alternative to the null hypothesis unspecified beyond "different from zero," or in the case of one-tailed tests, "less (greater) than zero." It is possible, however, to have a specific alternative to a null hypothesis, which can be composite with as many independent components as one likes. Expression of a specific alternative hypothesis moves one in the direction of using Bayesian statistics, and some researchers object to this on the grounds that it usually requires the introduction of subjectivism because of the need to assign prior probabilities to the hypotheses and a probability to the data conditional on the alternative hypothesis. A counter to this argument is that subjective judgment is required in conventional non-Bayesian hypothesis testing as well, but in this case it is not quite so apparent.

I. J. Good (1981/1983b), who sees himself as neither a Bayesian nor an anti-Bayesian, argued this way for making the alternative hypothesis precise and explicit:

I believe it is a feature of all sensible significance-test criteria that they are chosen with either a precise or, more often, a vague non-null hypothesis, in mind. In this respect "non-Bayesians" act somewhat like Bayesians. If a tail-area probability is small enough then it is worth while to try to make the non-null hypothesis less vague or even to make it precise, and the smaller the tail-area probability, the more worth while it is to make this attempt. . . . It is sometimes stated by Fisherians that the only hypothesis under consideration is the null hypothesis, but I am convinced that this is only a way of saying that the non-null hypothesis is vague, not that it does not exist at all. (pp. 138–140)

A similar point has been made by Rogers et al. (1993), who argued that it is not a small p value alone that determines a researcher’s postexperiment belief about the null hypothesis, but “a small p value with a known experimental manipulation after random assignment” (p. 560) that does so. As a rule (though not always), an experimenter has a definite idea of how a planned manipulation will affect the dependent variable(s) of interest, at least to the extent of the direction of the anticipated effect. Rogers et al.’s point is that this idea, along with the value of p, determines the
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The experimenter’s interpretation of the outcome, which from a Bayesian point of view is as it should be.

The overarching issue here is what constitutes appropriate statistical treatment of the data obtained in an experiment and justified interpretation of the outcome depends to no small degree on the experimenter’s preexperiment intentions and expectations. For this reason, experimenters should make explicit their preexisting expectations—alternative hypothesis(es) and its (their) predictions regarding data. This is not to suggest that unexpected patterns in data cannot be informative and should be ignored. However, what they provide are generally better viewed as clues to possible relationships that should be explored in further experimentation, not statistical justification for firm conclusions that such relationships exist.

Report p Values

Several writers, including Fisher (1956/1990), have advocated the reporting of actual p values as an alternative or complement to classifying results as either significant or not significant relative to a given α criterion (Abelson, 1995; Eysenck, 1960; Gibbons & Pratt, 1975; I. J. Good, 1981/1983b; Greenwald et al., 1996; Huberty, 1993; Schafer, 1993; Upton, 1992; Wang, 1993). Especially bothersome is the practice of reporting only whether the outcomes of tests attained statistical significance and not giving the values of the test statistics or, in some cases, means and standard deviations on which they were based (Meehl, 1978, 1997).

The reporting of p values makes the reporting of results a more objective process: When an investigator reports that p = .045 in one case and p = .055 in another, the reader can decide whether to consider one significant and the other not. On the other hand, reporting p values is not necessarily inconsistent with using an α criterion. There is the view that p values should be reported so readers can draw their own conclusions but that the conclusions drawn by experimenters should be based on predetermined criteria: Cortina and Dunlap (1997) expressed this view and argued that the latter restriction follows from the logic of theory testing, which involves determining whether predicted results are obtained.

Another argument for reporting actual p values is that it facilitates the aggregation of evidence across experiments by the use of meta-analyses. The practice is simplified by the fact that widely used statistical software packages yield specific values of p.

A minor problem associated with reporting actual p values is the fact that some software data-analysis packages return such values as .0000. Presumably this means that p = 0 to four decimal places; however, many readers object to the representation. A compromise is to revert to p < .0001 for all values of p less than .0001.

Summary of Defenses of NHST and Recommendations Regarding Its Use

Several writers have defended the use of NHST as an aid to the interpretation of experimental results and have argued that most if not all of the criticisms of it are more appropriately directed at ways in which it is sometimes misapplied and are not justified criticisms of NHST per se. Some defenders of NHST acknowledge that many users have misconceptions of it and consequently misapply it, but they argue that many of the criticisms that are leveled against the procedure are based on misconceptions or false beliefs as well.

Proponents of the use of NHST have also suggested several ways in which the effectiveness of its use might be enhanced. These suggestions include making greater use of non-nil null hypotheses, including range null hypotheses; being specific about alternatives to the null hypothesis; and reporting of actual p values rather than only classifying outcomes as either statistically significant or not statistically significant relative to a prespecified criterion.

Alternatives or Supplements to NHST

As noted early in this article, some of the NHST’s critics argue that this method of data analysis should be abandoned. Nobody, to my knowledge, even among its staunchest defenders has argued that NHST is the only type of analysis of data that one needs to do. In what follows, I note a variety of suggestions that have been made of ways either to supplement NHST or to replace it with alternative methods for evaluating the results of research.

Provide Some Indication of Variability or Precision of Measurement

Although variability due to measurement error is widely recognized as an important consideration in the evaluation of data obtained in nearly all psychological experiments, indications of it are lacking in many published reports of experimental results, and especially on the figures appearing in those reports. Estes (1997a) argued strongly for the inclusion of indications of variability on graphical representations
of data but cautioned that such indications should be accompanied by information regarding what data they are based on and how they were computed.

Some measures of variability (e.g., standard deviations, interquartile ranges) are simply descriptive of the data in hand and say nothing directly regarding the populations from which samples have been drawn. Others (e.g., standard errors of means and confidence intervals) are used to make inferences about the variability of populations and are subject to misconceptions and confusions not unlike those that pertain to the $p$ values of NHST.

The standard error of the mean is an estimator of the standard deviation of sample means around a population mean, and when used to bracket the population mean it indicates the range around the true mean within which about two thirds of the means of replicated random samples would be expected to fall. In actual experiments (simulations aside) population means are not known and standard error bars are usually drawn around sample means, and here they invite misinterpretation. They do not indicate the range around a sample mean within which about two thirds of the means of replicated random samples would fall. Given reasonable assumptions about the relationship between the standard error of the mean and the standard deviation of the population of sample means, about two thirds of the means of replicated random samples would fall within a range of about 1.4 times the standard error on either side of the sample mean (Estes, 1997a). Estes explained the common misinterpretation of standard errors and gave guidelines for computing and communicating measures of variability based on sample data for both independent-group and repeated-measures experimental designs.

Report Confidence Intervals Around Point Estimates

Among the indications of variability that might be used, confidence intervals deserve special attention because the reporting of them around point estimates has been advocated as an alternative, or adjunct, to NHST by many writers, including both critics and proponents of NHST (Bailer & Mosteller, 1988; Bolles & Messick, 1958; Cohen, 1990, 1994; Gardner & Altman, 1986; Gonzalez, 1994; Grant, 1962; Hunter, 1997; Jones, 1995; Loftus, 1991; Loftus & Masson, 1994; Meehl, 1997; Mulaik et al., 1997; Rozeboom, 1960; Schmidt, 1996; Serlin, 1993; Steiger & Fouladi, 1997; Tukey, 1991).

Although confidence intervals are probably most commonly reported for means or for differences between means, they can be computed for other statistics as well. Usually upper and lower interval bounds are equidistant from the estimated point value, but they are not always so (Darlington & Carlson, 1987). Steiger and Fouladi (1997) made a strong case for the advantages of confidence intervals and described a general method for computing them for variables for which they are seldom reported. Techniques for computing confidence intervals around means for between-subjects experimental designs are readily available; recently Loftus and Masson (1994) have described procedures for computing such intervals for within-subject designs. The fact that these cases require different computations suggests the need for care in reporting computational approaches along with the intervals computed (Estes, 1997a).

An attraction that is claimed for point estimates bracketed with confidence intervals is that confidence intervals are more informative than significance tests. One gets both an estimate of effect size and an indication of uncertainty as to its accuracy. These estimates can be especially helpful when there is interest in showing that the effect of some experimental treatment is nonexistent or small (e.g., an unwanted side effect of a new drug). Reichardt and Gollob (1997) suggested that attaching more importance to precision of estimates (e.g., narrowness of confidence intervals) in publication decisions could help address problems stemming from the overriding importance now attached to statistical significance. Hedges (1987) argued that something equivalent to the determination of confidence intervals is typically done in the physical sciences. (See also Meehl, 1967, in this regard.) Mulaik et al. (1997) noted that one reason that physical scientists do not use NHST so much is that they are not always testing hypotheses but rather trying to improve estimates of physical constants.

Some writers have cautioned that although confidence intervals can be very effective aids to the interpretation of data, they are subject to the same types of misconceptions and misuses as is NHST (Abelson, 1997a; Cortina & Dunlap, 1997; Frick, 1995b, 1996; Hayes, 1998). As Abelson (1997a) said: "Under the Law of Diffusion of Idiocy, every foolish application of significance testing will beget a corresponding foolish practice for confidence limits" (p. 13). Steiger and Fouladi (1997) warned that the proper use of confidence intervals requires assumptions about distributions, just as NHST does, and
It is easy to find discussions of confidence intervals that convey just this idea, which would be true if the first measure of the statistic happened to coincide with the population parameter and all subsequent samples were randomly drawn from the same population, but not otherwise. This misinterpretation may be reinforced by language such as the following that is sometimes found in statistics texts: "An alternative approach to estimation is to extend the concept of error bound to produce an interval of values that is likely to contain the true value of the parameter" (Bhattacharyya & Johnson, 1977, p. 243). The authors of this comment went on to point out that it is not correct to interpret a confidence interval of, say, 95% as an indication that the probability that the parameter of interest lies within the specified interval is .95, but it would not be surprising if the above comment itself is interpreted by readers in that way.

Another example comes from Elifson, Runyon, and Haber (1990), who after computing upper and lower limits for the 95% confidence interval for a sample mean of 108 and sample standard deviation of 15 said,

Having established the lower and upper limits as 101.82 and 114.18, respectively, we may now conclude: On the basis of our obtained mean and standard deviation, which were computed from scores drawn from a population in which the true mean is unknown, we assert that the population mean probably falls within the interval that we have established. (p. 367)

These authors went on to caution, "Since the population mean is a fixed value and does not have a distribution, our probability statements never refer to \( \mu \)." I find it difficult to see these two assertions as consistent.

Bhattacharyya and Johnson (1977) contended that given an appropriately computed confidence interval of \( x \% \) it is correct to say that one is \( x \% \) confident that the parameter of interest lies within the interval, but it is not correct to say that the probability is \( x \% \) that the parameter lies within the interval. The statement of confidence, in this view, rests on the expectation that for 95% of the samples of the same size drawn at random from the same population, confidence intervals computed in the same way will contain the value of the parameter, which is different from the belief that the probability is .95 that the parameter lies within the particular interval computed. (See also Darlington & Carlson, 1987; Elifson et al., 1990; Pruzek, 1997, regarding the same distinction.) This is a subtle distinction, even discounting the thorny question of what probability really means, and it is not hard to understand that with respect to the interpretation of confidence intervals confusion appears to reign.

Inasmuch as reporting confidence intervals has been urged by many parties on both sides of the debate about NHST, the infrequency of their appearance in published articles (Kirk, 1996) begs an explanation. The question is the more puzzling in view of the fact that the use of point estimates and confidence intervals ("error bands") predates the development of NHST (Oakes, 1986; Schmidt & Hunter, 1997). Cohen's (1994) surmise is that the reason they are not reported is that, at least when set at 95% to correspond to a .05 \( \alpha \) level, they typically are embarrassingly large.

Reichardt and Gollob (1997) ventured several "potential sources of resistance to the use of confidence intervals" (p. 277), which in abbreviated form are as follows: (a) convention (strength of tradition); (b) lack
of recognition of conditions in which confidence intervals are preferable to significance tests; (c) relative paucity of computer programs and formulas for computing confidence intervals; (d) the often disappointingly small size of parameter estimates—confidence intervals reveal this whereas significance tests do not; (e) the often disappointingly wide range of confidence intervals; (f) the absence of unique confidence intervals when no uniquely defined parameter is associated with a statistical test, which means computing a confidence interval may mean choosing among candidate indexes and defending the choice; (g) arguments against significance testing recognized to be fallacious that may, in some cases, damage the credibility of claims of the superiority of confidence intervals; and (h) some researchers' rejections of recommendations to abandon NHST (which sometimes accompany promotion of the use of confidence intervals) because they interpret them to be recommendations to abandon hypothesis testing more generally and they do not consider that to be desirable.

Estes (1997a) cautioned that because confidence intervals can be computed in different ways, the method of computation must be known in any particular case if confusion is to be avoided. (Methods of computation depend on such considerations as sample size, whether the standard deviation of the population is known, whether the distribution of interest is symmetric, and so on.) He recommended against showing "quasi" confidence intervals (intervals calculated from interaction terms rather than from within-cell means) on figures, on the grounds that showing them could invite misinterpretation. He noted, too, the importance of consistently using intervals of the same width (percentage) within a given research report.

Report Effect Size

Many writers have recommended that researchers statistically provide some indication of effect size either along with or in place of the results of statistical significance tests (Abelson, 1995; Carver, 1978, 1993; Cohen, 1988; Cook & Campbell, 1979; Fisher, 1925; Fleiss, 1969; Folger, 1989; Friedman, 1968; Glass, 1976; Guttmann, 1981; M. J. Harris, 1991; M. J. Harris & Rosenthal, 1985; Hurlburt, 1994; Katzner & Sodt, 1973; Loftus, 1993; Nelson et al., 1986; Schaefer, 1993; Schmidt, 1996; Snyder & Lawson, 1993; Thompson, 1994a, 1996). The reporting of effect size is required by some journal editors (Murphy, 1997; Thompson, 1994a); it is encouraged but not required by the Publication Manual of the American Psychological Association (4th ed., APA, 1994, p. 16).

The term effect size has an unfortunate ambiguity as applied to the results of experimentation. Its most straightforward connotation is that of the magnitude of some measure, such as the size of the difference between two means or the degree of association (covariation) between two variables. It can also suggest, however, the theoretical or practical impact of a finding. The former connotation is intended here and, I believe, in most discussions of effect size in the psychological literature, but the ambiguity is a source of confusion. (Sometimes a distinction is made between measures of effect size and measures of strength of relationship [e.g., Kirk, 1996], but for present purposes it will suffice to let the meaning of effect size be sufficiently broad to encompass both.) As Maher (1998) noted it is unfortunate that the word effect is used to denote a computation "that tells us nothing about the concrete effects produced by an intervention" (p. 211). Shaver (1991, 1993), who argued for another reason that the term should be result size, contended that this should always be reported and that the results of statistical significance tests and of power analyses should never be.

A variety of indicants of effect size have been proposed (Cliff, 1993; Cohen, 1962, 1977; Cohen & Cohen, 1983; Fleiss, 1969; Friedman, 1968; Hays, 1994; Hedges, 1981; Judd, McClelland, & Culhane, 1995; Maxwell, Camp, & Arvey, 1981; Rosenthal & Rubin, 1982; Snyder & Lawson, 1993; Tatsuoka, 1993). Several, including the actual magnitude of the observed effect (e.g., difference between means), a standardized magnitude (the actual magnitude normalized by the within-groups standard deviation), Pearson's r and \( r^2 \), and a measure of "causal efficacy" (the raw magnitude of effect divided by the magnitude of the variation in the independent variable) are discussed by Abelson (1995). The Publication Manual of the American Psychological Association (4th ed.; APA, 1994) mentions as common measures \( r^2, \eta^2, \omega^2, R^2, \phi^2, \) Cramer's V, Kendall's W, Cohen's d and \( \kappa \), Goodman and Kruskal's \( \lambda \) and \( \gamma \), Jacobson and Truax's (1991) proposed measure of clinical significance, and the multivariate Roy's \( \Omega \) and the Pillai-Bartlett \( V' \) (p. 18). Kirk (1996) listed almost all of these plus several others; however, on the basis of a survey of four APA journals, he noted that most of these measures are seldom if ever found in published reports.

Among the more widely used indicants is \( r^2 \) (sometimes called the coefficient of determination), which
shows the percentage of variance in a dependent variable accounted for by variance in an independent variable. Some writers have argued that $r$ is a better indicant than $r^2$, however, for many practical purposes (Nelson et al., 1986). Rosenthal and Rubin (1982) believed that the importance of effects are often underestimated when the value of $r^2$ is small especially perhaps in biomedical or clinical contexts.

Another widely used indicant of effect size is Cohen's (1962, 1988) $d$, which is the difference between means divided by the pooled within-groups standard deviation. Like the familiar $z$ score, $d$ expresses a difference in standard-deviation units. Such a measure has the advantage of facilitating comparisons of effect sizes across studies involving measurements on different scales.

Pointing out that statistical significance is a function of two factors—effect size and sampling error—Carver (1993) argued that reporting each of these measures is preferable to reporting a statistic such as $t$ that combines them.

Statistical significance tells us nothing directly relevant to whether the results we found are large or small, and it tells us nothing with respect to whether the sampling error is large or small. We can eliminate this problem by reporting both effect size and standard errors. (p. 291)

The reporting of effect size, however, is not totally free of problems. Which one of various possible effect-size estimates is most appropriate in a given context is not always apparent (Rosenthal, 1991), and opinions differ regarding the merits of specific possibilities (Crow, 1991; Gorsuch, 1991; McGraw, 1991; Parker, 1995; Rosenthal, 1991; Strahan, 1991). Moreover, the implications of effect-size estimates, like those of other statistics, need to be interpreted carefully. It is not safe to assume, for example, that the size of an effect obtained with one population, such as college students, will generalize readily to other populations, such as nonstudents (Sears, 1986). Furthermore, a large effect is no more of a guarantee of the theoretical or practical importance of a finding than is a small $p$ value resulting from a significance test (Chow, 1991; Shaver, 1985); a small effect may be very important in some contexts and a large one of little interest in others (Lewandowsky & Maybery, 1998; Prentice & Miller, 1992; Rosenthal, 1990; Rosenthal & Rubin, 1979). Finally, effect size should not be confused with strength of belief; as Abelson (1995) has pointed out, one may have a strong belief in a small effect or a weak belief in a large one.

The reporting of effect size may be seen as consistent with the use of NHST and an important complement to it (Hagen, 1998; Thompson, 1993). Robinson and Levin (1997) recommended the policy of first determining whether an effect is statistically improbable and then, only if it is, including in the reporting an indication of its size. Alternatively, as already noted, one may decide first the minimum effect size that would be large enough to be of interest and then use NHST to test the hypothesis that the effect is statistically significant only if it meets the size test (Fowler, 1985).

Rosnow and Rosenthal (1989b) maintained that effect sizes should be calculated whether or not $p$ values reach conventional levels of statistical significance. They noted that effect size tells the experimenter something that a $p$ value does not, and they suggested that such computations can be useful in determining sample sizes for subsequent experimentation. The standard reporting of effect size has the added attraction for some analysts of facilitating the use of meta-analytic techniques (Asher, 1993; Glass et al., 1981; M. J. Harris & Rosenthal, 1985; Mullen, 1989; Rosenthal, 1984). Despite urgings for the consistent reporting of effect size, it appears not to have become standard practice yet (Kirk, 1996; Thompson & Snyder, 1997, 1998). One practical impediment to the use of effect-size indicants as well as of confidence intervals may be poor understanding of them among researchers (Svyantek & Ekeberg, 1995).

A contrary view regarding the importance of reporting effect sizes has been given by Chow (1988, 1996), who argued that it can be important when an experimenter is interested in an experimental effect per se and the purpose of an experiment is to determine whether that effect is of sufficient size to have practical importance (utilitarian experiment), but that it is not relevant when the purpose of the experiment is to test an implication of an explanatory theory (theory-corroboration experiment). In the latter case, Chow (1996) claimed, the only relevant consideration is whether the obtained result is consistent with that predicted by the theory, within the criterion represented by the $\alpha$ level, and that an emphasis on effect size in this context can be misleading. Chow (1996) stressed that obtaining a result that is consistent in this statistical sense with the implications of a theory does not prove the theory true, but he argued that it does add incrementally to the theory’s tenability.
Use Power Analyses

It is widely agreed, I believe, that when one wants to conclude from the failure of a test to establish statistical significance that there is no effect or that any effect there might be is probably negligibly small, a power analysis should be done if possible (Meehl, 1991; Robinson & Levin, 1997; Schafer, 1993). Only if the power of the test was high should one treat the null hypothesis as true (or approximately true), and then only with sensitivity to the fact that even a test of great power does not prove it to be so.

One hypothesized reason for lack of attention to power by experiments is the difficulty of doing power analyses. This problem has been addressed by R. J. Harris and Quade (1992; R. J. Harris, 1997a) in their development of "minimally important difference significance," a method for calculating the sample size "that would be barely sufficient to yield statistical significance if your sample effect size is 'minimally important,'" that is, right on the borderline between having and not having practical or theoretical importance" (R. J. Harris, 1997a, p. 166). Tables for determining sample sizes that are appropriate for detecting effects of specified magnitude at specified levels of significance are given also by Cohen (1977) and Kraemer and Thiemann (1987). The burden is on the researcher, of course, to specify what constitutes a minimally important sample effect size.

Most of the discussion of power in the psychological literature has focused on the question of how large a sample must be in order for a given effect size to be likely to yield statistically significant results and on the researcher's problem of selecting a sufficiently large sample to detect an effect if there is one to detect. However, some writers have also argued the importance of ensuring that power is not so great that effects too small to be of interest will prove to be statistically significant: "It is important for researchers to consider the power of their experiments not only to detect the effects they seek, but also to avoid detecting trivially small effects" (Rossi, 1997, p. 184). If an experimenter is in the position of being interested in an effect only if it is relatively large and robust—which is to say, an effect that would be likely to be detected even with a relatively small sample—then high power is not a requirement and could even be undesirable. As is the case with so many aspects of the use of statistics in hypothesis testing or decision making more generally, the importance of power analysis depends on the experimenter's intentions, and its use requires some judgment.

Shaver (1993) has challenged the use of power analysis on the grounds that its primary purpose is to determine the sample size that is needed to yield statistically significant evidence of an effect of a given magnitude, and that this is pointless if statistical significance has little meaning, as he assumes.

If effect sizes are important because statistical significance (probability) is not an adequate indicator of magnitude (or much of anything else), why play the game of adjusting research specifications so that a statistically significant result can be obtained if a prespecified effect size is obtained? (p. 309)

Schmidt and Hunter (1997; Schmidt, 1996) have also argued that requiring researchers to use large enough samples to ensure high power is neither a practical nor desirable solution to the problems of NHST.

Like all tools in the statistical analyst's kit, power analysis has its limitations. How likely an experiment is to yield statistically significant results depends not only on the size of the effect (assuming one exists) and the size of the sample but also on the within-condition variability of the data. Within-condition variability can be influenced by many factors, but the more it can be limited by careful experimental control of extraneous variables, the more likely an effect of a given size is to show up as statistically significant, other things being equal; and power analysis is not sensitive to this fact. It should be borne in mind, however, that attempts to minimize within-group variability can have the effect of limiting the generality of findings (Pruzek, 1997); findings obtained with relatively homogeneous samples (e.g., college sophomores) do not necessarily generalize readily to more heterogeneous populations (e.g., the general public).

Use Three-Outcome Tests

Some researchers have argued that the traditional one- and two-tailed significance tests should be replaced with three-outcome tests (Bohrer, 1979; R. J. Harris, 1994, 1997a, 1997b; Kaiser, 1960). One-tailed tests do not permit one to determine that an effect is statistically significant in the direction opposite that hypothesized, and two-tailed tests do not justify a conclusion about the direction of an effect (although R. J. Harris, 1997a, pointed out that this does not stop users of two-tailed tests from drawing such conclusions). Some investigators have proposed modifications of conventional NHST (e.g., split-tailed tests) that would permit rejection of the null hypothesis as a consequence of a difference in the direction opposite
the predicted one (Braver, 1975; Nosanchuk, 1978), but such tests appear not to be widely used.

A three-alternative test can be thought of as equivalent to two one-tailed tests (one in each direction) in combination (Kaiser, 1960; Shaffer, 1972). With such a test an experiment can lead to the decision that mean₁ is smaller than mean₂, that mean₁ is larger than mean₂, or that the direction of the difference (if any) between their sizes is indeterminate.

R. J. Harris (1997a, 1997b) argued that many of the misinterpretations of NHST are fostered by its presentation as allowing a choice between two hypotheses and that those misinterpretations could be addressed by the use of three-alternative tests. One conjecture, for example, is that people who are exposed to three-alternative NHST will be less likely to treat nonrejection of H₀ as acceptance of it. Unlike two-valued tests, a three-valued test makes it possible for one to obtain statistically significant evidence against one's research hypothesis.

Three-alternative tests provide a finer grained partitioning of outcomes than do two-alternative tests, but they do not avoid most of the problems associated with the latter. Moreover, despite the fact that their use was promoted as early as 1960, they have not gained much popularity among researchers (R. J. Harris, 1997a; Hunter, 1997).

Use Parameter-Estimation and Model-Fitting Techniques

Among alternatives to NHST are those of parameter estimation and model fitting. In these cases, one is not asking whether two samples differ statistically in some specified way or by more than some specified amount; rather, one is attempting to estimate the value of some parameter of a population or to determine how well a model predicts the value of one or more experimental variables.

One can argue that parameter estimation should be a method of choice if one’s objective is, to use a distinction made by Grant (1962), not that of deciding whether to accept or reject a finished theory but that of working, long-term, on the improvement of theory. Given the latter objective, “our statistical tactics can be greatly improved by shifting emphasis away from over-all hypothesis testing in the direction of statistical estimation” (p. 57).

Granaas (1998) argued that model fitting is not only more powerful than NHST but also simpler to learn and to use. An attractive feature of the approach is that model fitting can proceed in a closed-loop fash-

ion: An existing model is retained until one that fits the data better comes along, at which point it is replaced by the better fitting one. A variety of measures for evaluating goodness of fit have been developed, including measures of “error” between predicted and obtained results and percentage of variance accounted for by a model. (Although parameter estimation and model fitting are often seen as alternatives to NHST, it should be noted that a form of NHST plays a role in some measures of goodness of fit [T. D. Wickens, 1989].)

Goodness of fit is not the only criterion for evaluating a model. One can always define a model that fits a set of data perfectly by giving it a sufficiently large number of parameters, but the more parameters a model has, the less generalizable it is likely to be; and other things being equal, the simpler the model, the better from a scientific point of view. Other criteria that should be taken into consideration in evaluating a model or selecting among competing ones include explanatory power, plausibility, and internal consistency (Myung & Pitt, 1997).

Demonstrate Replicability by Replicating

Replicability is generally recognized as the sine qua non of an experimental finding that is to be considered scientific. However, as already noted, replication has more than one connotation. Lykken (1968) distinguished three: exact, or literal, replication of the conditions of an earlier study to the degree possible; operational replication, in which an attempt is made to reproduce what the experimenter perceives to be the main aspects of the earlier experimental situation; and constructive replication, in which the same constructs or relationships are investigated in a procedurally different way.

Experiments that are literal replications of previously published experiments are very seldom published—I do not believe I have ever seen one. Others who have done systematic searches for examples of them confirm that they are rare (Mahoney, 1976; Sterling, 1959). It is easy to identify factors that could contribute to the paucity of exact replication studies. Most researchers would probably not find experiments designed only to replicate either their own results or those of other researchers especially challenging or interesting. PhD committees generally expect more from dissertations than the replication of someone else’s findings. Evidence suggests that manuscripts that report only replication experiments are
paring the results of analyses on the resulting data have been proposed. These typically involve results short of conducting separate replication experiments; Humphreys, 1980).

original experiment (i.e., the nonsignificant effect is ordinal similarity to the attempting replication when an experiment has yielded an effect can decrease the probability that the original finding was a Type I error, provided the outcome of the attempt at replication will be successful also, nor does it prove the theory that predicted the effect is necessarily true; it does justifiably increase one’s confidence that further replication is obtainable, however, and lends some credence to theories that predict the effect as well.

Use the Bayesian Approach to Hypothesis Evaluation

The replacement or complementation of NHST with the use of likelihood ratios and the estimation of posterior probabilities by application of Bayes’s rule has been proposed by many writers (Edwards, 1965; Edwards et al., 1963; Gelman, Carlin, Stern, & Rubin, 1995; I. J. Good, 1981; Greenwald, 1975; Lindley, 1984; Rindskopf, 1997; Rouanet, 1996; Rozeboom, 1960; Rubin, 1978). In theory, this possibility has much to recommend it. A strong argument in favor of Bayesian hypothesis evaluation is that it permits evidence to strengthen either the null hypothesis or its alternative(s). Moreover, as Rouanet (1996) has said, a Bayesian analysis can lead either to the conclusion that the probability is high that a population effect is large or to the conclusion that the probability is high that a population effect is small, either of which could be of theoretical interest. (Rouanet, 1996, 1998, supported the idea of using Bayesian procedures to complement rather than replace NHST.)

These advantages are in contrast to classical NHST as conventionally used, which in Edwards et al.’s (1963) view, leaves the null hypothesis when it is not rejected “in a kind of limbo of suspended disbelief” (p. 235). Given a set of competing models of some process of interest, if one can specify for each model in the set a prior probability and the probability of the experimental outcome conditional on that model being true, then one can use Bayes’s rule to compute the probability of each model conditional on the outcome and determine the one for which the (posterior) probability is the largest.

Another major advantage of a Bayesian approach to data evaluation is that it constitutes a procedure for cumulating the effects of evidence across studies over time. In particular, the formalism allows for the posterior probabilities of one set of studies to be the priors for a subsequent one; at least in theory the process can be iterated indefinitely. As Pruzek (1997) pointed out, because of this fact sequences of studies analyzed within a Bayesian framework could sometimes obviate the need for meta-analysis. Classical
statistical methods do not provide for the incorporation of prior information in analyses; they generally treat the data from each experiment in isolation.

Perhaps many investigators who use classical NHST would hold that a Bayesian approach to hypothesis evaluation is to be preferred when the information that is necessary to apply Bayes's rule is known or can be plausibly assumed, but would contend that this condition is seldom met. The most common problem that arises is the difficulty of specifying prior probabilities (Frick, 1996). A standard default assumption in Bayesian analysis is that the hypotheses under consideration are all equally probable a priori.

Although less often discussed, estimating a value for $p(D \mid H_{A})$ is also a problem. As we have noted, a criticism of NHST is that it considers only the probability of the observed result given the distribution of possibilities assumed by the null hypothesis and that it does not consider the likelihood of the result given the distribution assumed by a specific alternative hypothesis. W. Wilson et al. (1967) acknowledged the preference of the use of likelihood ratios when the information needed to apply them—in particular the value of $p(D \mid H_{A})$—is available, but they defended the use of NHST when it is not:

> If we have a clearly defined alternative, and we can say that "reality" must be one or the other, then we can justify a likelihood ratio or some similar procedure. If we do not have such alternative models, we cannot invent them to avoid a theoretical bias. (p. 191)

Palm (1998) took a similar position.

Many researchers have objected to the use of a Bayesian approach to hypothesis evaluation on philosophical grounds. Fisher (1935), for example, rejected it on the grounds that it (sometimes) regarded mathematical probability as a reflection of psychological tendencies—beliefs—as opposed to an objective measure derived from observable frequencies, and he considered theorems involving such subjective entities to be useless for scientific purposes. This opinion represents one side of a long-standing controversy about what probability "really means." Opponents of the use of Bayesian analysis when there is no objective basis for assigning prior probabilities point out that different people will arrive at different conclusions, reflecting differences in their individual prior beliefs. They note, too, that when priors must be produced objectively it is not always clear that they are any more credible than subjectively estimated posterior probabilities would be (Oakes, 1986). The application of Bayesian techniques to the analysis of experimental data has proved to be at least as controversial among researchers and analysts as has NHST.

NHST and Bayesian analysis are sometimes contrasted as mutually exclusive approaches to the evaluation of evidence, and arguments are couched in such a way as to suggest that any positive (negative) statement that can be made with respect to one is necessarily a negative (positive) reflection on the other. In truth, it is not necessary to see the situation this way. It is possible to believe that each has strengths and limitations, and that which is preferred depends on the specifics of the situation in which one or the other is to be applied. Recent tutorial presentations of Bayesian methods include Lewis (1993), Winkler (1993), Bernado and Smith (1994), and Robert (1994).

**Summary of Alternatives or Supplements to NHST**

Each of the suggested alternatives or supplements to NHST mentioned above has its proponents. Expositions and defenses of many of them can be found in a recent collection of articles on the subject (Harlow, Mulai, & Steiger, 1997; for a review, see Nickerson, 1999). In an overview chapter Harlow (1997) listed eight practices of scientific inference suggested by the contributors to the collection. One of the practices (the least strongly endorsed by the contributors in the aggregate) was the making of dichotomous decisions with NHST; each of the following additional seven could be considered an alternative or complement to NHST: (a) Assess strong theories with careful thinking and sound judgment; (b) focus on estimation and the width of an appropriate confidence interval; (c) calculate effect sizes and power; (d) evaluate how well a model approximates the data, without necessarily attending to issues of statistical significance; (e) make null and alternative hypotheses very specific (i.e., a particular nonzero value) and realistic; (f) replicate results in independent studies or quantitatively summarize using meta-analysis; (g) use Bayesian methods of inference.

As to how NHST compares with the other seven practices, Harlow (1997) had this to say:

This method of focusing on a dichotomous decision would contribute little to the development of strong theories or sound judgment; lacks the precision of either confidence intervals, effect sizes, or power calculations; is less informative than goodness of approximation assessment or the use of specific, realistic, and nonzero hypotheses; and is less thorough than either replication,
meta-analysis, or Bayesian inference. In sum, the overriding view on this issue is that NHST may be overused and unproductive, particularly when used as simply a dichotomous decision rule. (p. 12)

There can be little doubt that NHST has many critics and that there is general agreement among them that the method, especially when used as the only indicant of scientific credibility, is seriously flawed. Harlow's (1997) negative summary of its weaknesses was intended, I assume, to represent the overriding, though not unanimous, view among the contributors to Harlow et al.'s (1997) volume. The extent to which that view prevails among the research community more generally is not so clear.

**Other Recommendations**

In addition to recommendations pertaining directly to the use of NHST or regarding specific alternative or complementary approaches to data analysis, a number of more general recommendations have been made or could be made. I mention a few here that I think are especially noteworthy.

*Make a Clear Distinction Between the Substantive Question(s) of Interest and the Statistical Hypothesis(es) to Be Tested*

Researchers should distinguish statistical from epistemic questions, that is, when they are making an inference concerning a parameter (point value, range, slope, sign) from a statistic versus when they are appraising the verisimilitude of a substantive theory (causal, compositional, or historical) on the basis of the inferred parameters. (Meehl, 1997, p. 422)

It seems to me that this recommendation by Meehl, if followed carefully, would go a long way toward solving many of the problems associated with NHST. This surmise is based on the assumption that at the root of many of these problems is either a confusion between epistemic and statistical hypotheses or a focus on the latter type to the neglect of the former.

*Make NHST Subsidiary to Other Considerations in Evaluating Data*

Carver (1978, 1993) argued that testing for statistical significance before considering whether the data are generally supportive of the experimental hypothesis of interest is a corruption of the scientific method and that the rule should be to always interpret the results with respect to the data first and do statistical significance testing only secondarily. As to the argument that one may have to test for statistical significance in order to see whether there is any effect worth further consideration, Carver had little patience with it: "A study with results that cannot be meaningfully interpreted without looking at the p values is a poorly designed study" (1978, p. 394).

A not dissimilar sentiment has been expressed by I. J. Good (1981/1983b): "Personally I think that Rule 1 in the analysis of data is 'look at the data!' " (p. 138). Rozeboom (1997) encapsulated a closely related idea in what he called "the statistical relevance precept," according to which one should think through what one would want to make of a finding if one could obtain data with no sampling error. "You have nothing to gain from concern for a statistic's sampling uncertainty (save to oblique colleagues who want it) if you have little idea of what to do with its population value were you to know that" (p. 305). Rouanet (1996), who argued strongly for the use of a Bayesian approach to data analysis, also contended that one should consider first whether an effect is sufficiently large (or sufficiently small) to be of interest if it held for the relevant population as well as for the sample before doing any statistical analyses. "If this is not the case, no corresponding inferential conclusion is reachable. If this is the case, the aim ascribed to inferential procedures should be to extend descriptive conclusions to the population, allowing for sampling fluctuations" (p. 150).

Sohn (1998b) recommended that, at least in the case of atheoretical research, findings should be taken seriously only when they can be obtained consistently with individuals. In effect this means they should be discernible without the help of inferential statistics:

In the context of atheoretical research... effects need to be so robust that they are discernible in the individual organism so consistently that there is general agreement that the treatment is producing an effect. In such a case, significance tests are supererogatory. (Sohn, 1998b, p. 307)

Sohn (1998a) expressed the opinion that in psychology "findings that are not obviously discernible in nearly every instance are likely to be ignored. And if they are not ignored, they typically become a source of controversy" (p. 334).

What makes an experimental result worth publishing is a matter of opinion. I suspect there are few researchers who would argue that statistical significance alone suffices, but one must wonder about the extent to which fixation on NHST to the exclusion of
other considerations has contributed to the “indiscriminate cataloguing of trivial effects,” which W. Wilson et al. (1967) referred to 30 years ago as “a major problem in psychology today” (p. 195). More recently, Abelson (1997a, 1997b) has noted that significance tests are sometimes applied in unthinking ways to test differences that have little or no relevance to the question of interest or that are apparent without the benefit of the test.

The indiscriminate use of NHST has been criticized in even stronger terms by Edwards (1965), who has commented on what he considers to be the violent bias of classical hypothesis testing against the null hypothesis:

This violent bias of classical procedures is not an unmitigated disaster. Many null hypotheses tested by classical procedures are scientifically preposterous, not worthy of a moment’s credence even as approximations. If a hypothesis is preposterous to start with, no amount of bias against it can be too great. On the other hand, if it is preposterous to start with, why test it? (p. 402)

Sometimes statistics are used to compensate for poorly controlled experimentation. Variables are varied unsystematically, and little attention is given to the possibility of artifactual effects, the assumption being that statistical analyses will sort everything out. However, it is precisely where statistics are most needed to make sense of noisy data that significance tests lend themselves most readily to misinterpretation and misuse. As W. Wilson et al. (1967) said: “It makes relatively little difference which approach you use in precise experimentation. With great precision, you cannot go too far wrong” (p. 194). Conversely, given sloppy experimentation statistics are at least as likely to obfuscate as to clarify.

To keep NHST and other statistical procedures in perspective, it is well to also bear in mind that much scientifically solid and exceptionally influential psychological research has been done with relatively little, if any, use of statistics. The names of James, Bartlett, Piaget, Skinner, and Wertheimer come immediately to mind. Moreover, even effects that are statistically significant at a small value of \( p \)—and are large—are not guaranteed to be interesting and important; these aspects can be determined only relative to some value system that is beyond the province of statistics. A single-minded concern for statistical significance untempered by other considerations does not serve the best interests of any scientific field.

Consider the Intended User of the Research Findings

I have appropriated this recommendation from C. D. Wickens (1998) because it seems to me an obviously correct and important but neglected point. I suspect that most participants in the debate about NHST assume that the primary audience for research reports is other researchers, but potential users of the results of applied research include people other than researchers. What is most helpful by way of statistical analyses and statistical reporting for researchers may not necessarily be what is most helpful for practitioners. For example, while the conservative policy of guarding against Type I error at the expense of allowing a high incidence of Type II error may be in keeping with the idea that novel hypotheses should not be accepted or old ones discarded except on “beyond-a-reasonable-doubt” evidence, when a decision must be made between two possible approaches to a practical problem and the costs of both possible errors are comparable, a “preponderance-of-evidence” criterion may be more appropriate.

Understand (Perhaps Even Explain) Statistical Approach and Rationale for It

Whatever approach one takes to hypothesis evaluation, it seems a good idea to understand why one is taking that approach. It is at least a plausible conjecture that many users of statistical tests lack a deep conceptual grasp of the logic of those tests, the conditions that legitimize their use, and the conclusions they justify. The main effects of such lack of understanding are likely to be misapplication of tests and the drawing of unsubstantiated conclusions from their outcomes; and they can only be magnified by the ready availability of software that makes statistical tests—whether appropriate or not—trivially easy to perform.

Not only is understanding of the statistical tests one uses important, sometimes it may be desirable to explain the rationale for one’s statistical approach. This may mean stating any important assumptions that will not be obvious to the reader. If, for example, one is using a point null hypothesis as a convenient proxy for a range null, it would help to make this clear and to specify the range. It also means being explicit about how a statistic was computed when more than one method is possible and the different methods yield different results.
Choose Language Carefully

Undoubtedly some of the misconceptions about NHST are maintained and reinforced by the use of less-than-precise language in discussions of NHST and the results of such testing. The word *significance* invites misconception in the statistical context because of its everyday connotation of importance. Some writers have suggested discontinuing use of the word to convey statistical meaning—Chow (1996), for example, suggested replacing it with *nonchance*—but it seems unlikely that the term will disappear from psychologists’ vocabularies anytime soon. Some writers use *reliable* in lieu of *statistically significant*, but this could be interpreted as assuredly replicable.

Reporting statistical significance as simply "significance" undoubtedly contributes to the confusion between statistical significance and importance from a theoretical or practical point of view. This is not to suggest that one needs slavishly to qualify every instance of reporting the results of a statistical significance test with the word *statistical*, which can become tediously repetitive in some cases, but it is to suggest that one should be careful to make it clear that statistical significance—as opposed to theoretical or practical significance—is intended whenever there is a good chance of misinterpretation.

Careful use of language is especially important when speaking of conditional probabilities having to do with NHST. Reference to the $\alpha$ level as "the probability of Type I error" without making clear that it is the probability of rejecting $H_0$ conditional on $H_0$ being true illustrates the point, as does reference to $\beta$ as "the probability of Type II error" without explicitly noting that $\beta$ is the probability of failing to reject $H_0$ conditional on $H_0$ being false. There are many other possibilities for confusion arising from failure to note the conditional status of probabilities, some of which have been mentioned in preceding sections of this article.

After writing the preceding paragraph, I checked some definitions in the fifth edition of a statistics text of some longevity (Witte & Witte, 1997) that happened to be within reach. I found the following statements: "Alpha ($\alpha$): The probability of a type I error, that is, the probability of rejecting a true null hypothesis" (p. 251). "Beta ($\beta$): The probability of a type II error, that is, the probability of retaining a false null hypothesis" (p. 253). "Power ($1-\beta$): The probability of detecting a particular effect" (p. 258). Each of the probabilities mentioned is a conditional probability. I am sure that Witte and Witte understand this and perhaps a careful reading of the entire presentation in the text would make the conditional status clear, but the definitions lend themselves to misinterpretation.

Consider Aggregate Evidence

NHST is usually done with respect to the outcome of a single experiment or the manipulation of a single experimental variable. As has been pointed out many times, few psychologists would be willing to decide on the truth or falsity of general tenability of a hypothesis on the basis of the outcome of a single experiment. What seems to happen is that researchers are persuaded of the tenability of a hypothesis by the accumulation of evidence for or against it over many different experiments. The impact of the outcome of a single experiment depends in part on the prior tenability of the hypothesis because of evidence in hand before the experiment was done. According to a Bayesian view, this is as it should be.

The use of meta-analytic techniques has been urged by many as a structured method for considering the implications of the results of sets of experiments in the aggregate (Cooper, 1979; Cooper & Rosenthal, 1980; Eddy, Hasselblad, & Shachter, 1992; Glass, 1976; Glass & Kliegl, 1983; Hunter & Schmidt, 1990; Hunter, Schmidt, & Jackson, 1982; Schmidt, 1992, 1996). Meta-analysis has the advantage, its proponents argue, that it provides a means of extracting useful information even from the results of experiments that have not been statistically significant on their own. Even Mulai et al. (1997), who strongly defended the use of NHST, recommend that in evaluating studies journal editors consider the potential of their data being useful in combination with those of other studies in meta-analyses. Schmidt and Hunter (1997) argued that single studies do not contain enough information to be decisive with respect to hypotheses and that it is only by combining the findings of individual studies through the use of meta-analysis that dependable scientific conclusions can be reached. They went so far as to suggest that

from the point of view of the goal of optimally advancing the cumulative scientific knowledge, it is best for individual researchers to present point estimates and confidence intervals and refrain from attempting to draw final conclusions about research hypotheses. These will emerge from later meta-analyses. (p. 52)

Without denying the importance of analytic techniques for aggregating the results of a large number of
experiments, I believe that individual experiments and their stand-alone results will continue to play a critical role in the advance of psychological knowledge. I can do no better than quote Abelson (1997b) on this point: "Even though a single study cannot strictly prove anything, it can challenge, provoke, irritate, or inspire further research to generalize, elaborate, clarify, or to debunk the claims of the single study" (p. 124). We need to also bear in mind that it is possible for knowledge to cumulate without the aid of specific meta-analytic techniques; it has done so in the physical sciences quite effectively for a long time. Moreover, meta-analysis is not entirely free of problems and offers its own set of possibilities for misuse (Chow, 1996; Erwin, 1997; Gallo, 1978; Knight, Fabes, & Higgins, 1996; Lepper, 1995; Leviton & Cook, 1981; Shapiro, 1997; Sohn, 1996; G. T. Wilson & Rachman, 1983).

**Recognize the Provisional Nature of Hypotheses**

In their defense of NHST, Mulaik et al. (1997) emphasized—following Fisher (1935)—the importance of recognizing the tenability or defeasibility of scientific hypotheses or generalizations. All hypotheses, no matter what the evidence respecting them at any given time, are subject to modification as a consequence of further information that is relevant to them.

**Conclusion**

NHST has been and is a controversial method of extracting information from experimental data and of guiding the formation of scientific conclusions. As Meehl (1997) said:

> Competent scholars persist in strong disagreement, ranging from some who think $H_0$-testing is pretty much all right as practiced, to others who think it is never appropriate. Most critics fall somewhere between these extremes, and they differ among themselves as to their main reasons for complaint. (p. 421)

"Strong disagreement" does not quite capture the intensity of some of the contributions to the debate. Although many of the participants have stated their positions objectively and gracefully, I have been struck with the stridency of the attacks by some on views that oppose their own. NHST has been described as "thoroughly discredited," a "perversion of the scientific method," a "bone-headedly misguided procedure," "grotesquely fallacious," a "disaster," and "mindless," among other things. Positions, pro or con, have been labeled "absurd," "senseless," "nonsensical," "ridiculous," and "silly." The surety of the pronouncements of some participants on both sides of the debate is remarkable.

One hypothesis worth entertaining about the controversy is that it is a tempest in a teapot. (I am reminded of a comment by Kac, 1964: "Whatever your views and beliefs on randomness—and they are more likely than not untenable—no great harm will come to you." Perhaps a case could be made for this possibility. What is the great harm if many people who use NHST believe that $p$ is the probability that the null hypothesis is true, or that a small $p$ is evidence of replicability, or that $\alpha$ is the probability that if one has rejected the null hypothesis one has made a Type I error? Claims to the contrary notwithstanding, there is room for doubt as to whether acquisition of psychological knowledge through experimentation has been greatly impeded by the prevalence of such beliefs or by any of the many other shortcomings of NHST that have been ably identified by its critics.

Moreover, evidence suggests that the confidence psychologists place in experimental findings tends to vary with statistical test outcomes in intuitively reasonable ways. Confidence generally varies inversely with $p$ value and directly with effect size and sample size (Beauchamp & May, 1964; Nelson et al., 1986; Rosenthal & Gaito, 1963, 1964). Confidence in a finding is also increased as a consequence of successful replication (Nelson et al., 1986). Despite the many problems associated with NHST, these predilections should facilitate orderly application of research results to the advance of psychological theory. Most importantly, under assumptions that may be valid for most experiments, a small value of $p$ is indeed indicative of a not-quite-so-small value of $p(H_0 | D)$. Perhaps it is the case, as Rindskopf (1997) has argued, that null hypothesis tests are still used because "they are testing approximately the right thing under many real circumstances, even though most researchers do not know the rationale" (p. 321).

On the other hand, as noted at the outset, NHST is arguably the most widely used method of analysis of data collected in psychological experiments and has been so for a long time. If it is misunderstood by many of its users in as many ways as its critics claim, this is an embarrassment for the field. A minimal goal for experimental psychology should be to attempt to achieve a better understanding among researchers of the approach, of its strengths and limitations, of the
various objections that have been raised against it, and of the assumptions that are necessary to justify specific conclusions that can be drawn from its results.

The situation is not simple—it is confused and confusing—and a nonsuperficial understanding of the issues requires a considerable investment of time and effort. Having made a bit of an effort in this direction myself, one conclusion that I have come to is that it is not necessarily the case that the more one learns about NHST, the clearer its proper role in psychological research becomes; a more likely consequence of learning, in my view, is the discovery that some of the principles and relationships one had considered well-established or that one had taken for granted are not beyond dispute. One finds conflicting opinions strongly held by knowledgeable people on various aspects of the topic, and it is not always easy to be sure of what one’s own should be. However, there is little virtue in being confident of one’s opinions when the confidence depends on not being aware of reasoned alternatives that exist.

The debate about NHST has its roots in unresolved disagreements among major contributors to the development of theories of inferential statistics on which modern approaches are based. Gigerenzer et al. (1989) have reviewed in considerable detail the controversy between R. A. Fisher on the one hand and Jerzy Neyman and Egon Pearson on the other as well as the disagreements between both of these views and those of the followers of Thomas Bayes. They noted the remarkable fact that little hint of the historical and ongoing controversy is to be found in most textbooks that are used to teach NHST to its potential users. The resulting lack of an accurate historical perspective and understanding of the complexity and sometimes controversial philosophical foundations of various approaches to statistical inference may go a long way toward explaining the apparent ease with which statistical tests are misused and misinterpreted.

On one point most writers agree: NHST cannot be done mechanically without running the risk of obtaining nonsensical results; human judgment must be an integral, and controlling, aspect of the process. Abelson (1997a) made this point forcefully:

Whatever else is done about null-hypothesis tests, let us stop viewing statistical analysis as a sanctification process. We are awash in a sea of uncertainty, caused by a flood tide of sampling and measurement errors, and there are no objective procedures that avoid human judgment and guarantee correct interpretations of results. (p. 13)

To the extent that the motivation to ban NHST stems from a desire to rid psychology or psychologists of misconceptions of the sort reviewed in the first part of this article, there is little reason to believe that such a ban would have the desired effect because the proposed alternatives to NHST lend themselves to misinterpretations and misuses as well. As R. J. Harris (1997a) said: “Banning significance tests is clearly not going to guarantee that misunderstandings of the role of sampling error and the strength of evidence required to establish the validity of a null hypothesis won’t continue to bedevil us” (p. 160). NHST surely has warts, but so do all the alternatives.

The indispensability of human judgment in the interpretation of the results of research, including the results of statistical tests, has been stressed by many (Berger & Berry, 1988; Browne & Cudek, 1992; Cohen, 1994; Cortina & Dunlap, 1997; Falk & Greenbaum, 1995; Gigerenzer, 1993; Huberty & Morris, 1988; Malgady, 1998). However, judicious use of statistical tests presupposes a level of mathematical sophistication that the training of social scientists often fails to achieve, and lack of this sophistication coupled with the ready availability of statistical software more or less ensures some inappropriate applications of statistical tests and unreasoned uses of their results (Estes, 1997b).

Arguments regarding the appropriateness of NHST will surely continue. Most Bayesians will undoubtedly argue against it, and many non-Bayesians will continue to use it; but this split is too simple. There are people who see the advantages of Bayesian analysis when the information that it requires is readily at hand who are not ready to declare NHST defunct. I. J. Good (1981/1983b) has put himself in this camp:

I personally am in favor of a Bayes/non-Bayes compromise or synthesis. Partly for the sake of communication with other statisticians who are in the habit of using tail-area probabilities, I believe it is often convenient to use them especially when it is difficult to estimate a Bayes factor. But caution should be observed when the samples are very large if the tail-area probability is not extremely small. (p. 143)

McGrath (1998) has argued that it is not a question of whether NHST is useless but rather whether there is something better. This is a question that should be asked, in my view, whenever one is attempting to decide how to extract information from data, and it applies not only to NHST but to any other approach one might consider using. There are no statistical pro-
procedures that can safely be used without thought as to their appropriateness to the situation or the question of whether there are more effective approaches available. Statistical methods should facilitate good thinking, and only to the degree that they do so are they being used well. When applied unthinkingly in cookbook fashion and without awareness of their limitations and of the assumptions that are needed to justify conclusions drawn from them, they can get in the way of productive reasoning; when used judiciously, with cognizance of their limitations, they can be very helpful in making sense of data and determining what conclusions are justified. NHST, like other statistical tools, can be applied in inappropriate and counterproductive ways—easily when its rules of application or its products are not well understood—but it can also be an effective aid to data interpretation when used appropriately as an adjunct to good experimental design and in conjunction with other methods of extracting information from noisy data.

Finally, in trying to assess the merits of NHST and other approaches to the evaluation of hypotheses, it is well to bear in mind that nothing of importance in psychology has ever been decided on the basis of the outcome of a single statistical significance test. Psychological knowledge is acquired, as is knowledge in other fields, as a consequence of the cumulative effect of many experiments and nonexperimental observations as well. It is the preponderance of evidence gathered from many sources and over an extended period of time that determines the degree of credibility that is given to hypotheses, models, and theories.

References


Abelson, R. P. (1997b). A retrospective on the significance test ban of 1999 (if there were no significance tests, they would be invented). In L. L. Harlow, S. A. Mulaik, & J. H. Steiger (Eds.), *What if there were no significance tests?* (pp. 117-141). Hillsdale, NJ: Erlbaum.


Bohrer, R. (1979). Multiple three-decision rules for para-


Cox, D. R. (1958). Some problems connected with statisti-
null hypothesis significance testing

DeGroot, M. H. (1973). Doing what comes naturally: Inter-

dawes, r. m. (1988). Rational choice in an uncertain

behavioral sta-

darlington, r. b., & carlson, p. m. (1987). behavioral psy-


eddy, d. m. (1982). probabilistic reasoning in clinical medicine: Problems and opportunities. in d. kahneman, p. slovic, & a. tversky (eds.), judgment under uncer-


NULL HYPOTHESIS SIGNIFICANCE TESTING


Harris, R. J. (1997b). Significance tests have their place. Psychological Science, 8, 8–11.


Loftus, G. R. (1996). Psychology will be a much better science when we change the way we analyze data. *Current Directions in Psychological Science, 5*, 161–171.


Meehl, P. E. (1990a). Appraising and amending theories:
The strategy of Lakatosian defense and two principles that warrant it. *Psychological Inquiry*, 1, 108–141.


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