Problem Solving
Chapter 12

Some Questions to Consider

- What makes a problem hard?
- Is there anything special about problems that seem to be solved in a flash of “insight”?
- How can analogies be used to help solve problems?
- How do experts in a field solve problems differently than nonexperts?

What Is a Problem?

- Obstacle between a present state and a goal
- Not immediately obvious how to get around the obstacle
How do we overcome obstacles?

Gestalt Approach

- Representing a problem in the mind
- Restructuring: changes the problem's representation
- Köhler’s “circle” problem
- Chimps and obstacles to food
Gestalt Approach

- Representing a problem in the mind
- Restructuring: changes the problem's representation
- Köhler’s “circle” problem

The circle problem

Problem: If the length of the circle's radius is \( r \), what is the length of line \( x \)?
Insight in Problem-Solving

- Sudden realization of a problem’s solution
- Metcalfe and Wiebe (1987)
  - Insight problems solved suddenly (“Aha!”)
  - Noninsight problems solved gradually
- Think-aloud protocol
  - Say aloud what one is thinking (thoughts, not actions)
  - Shift in how one perceives elements of a problem

Insight Problems

(a) Triangle problem. (b) Chain problem.

Insight in Problem-Solving

- Metcalfe and Wiebe (1987)
  - Insight: triangle problem, chain problem
  - Noninsight: algebra
  - Warmth judgments every 15 seconds
- Insight problems solved suddenly
- Noninsight problems solved gradually
Attach the candle to the wall so it will burn without dripping wax.

Tie the two strings together.
Obstacles to Problem Solving

- **Functional fixedness**: restricting use of an object to its familiar functions
- Candle problem: seeing box as a container inhibited using it as a support
- Two-string problem: function of pliers gets in the way of seeing them as a weight

Obstacles to Problem Solving

- Situationally produced *mental set*
- Situation influences approach to problem
- Water-jug problem: given mental set inhibited participants from using simpler solution
- Functional fixedness: the elevator riddle

Luchins’s (1942) water-jug problem

<table>
<thead>
<tr>
<th>Problem</th>
<th>Jug A</th>
<th>Jug B</th>
<th>Jug C</th>
<th>Desired quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
<td>127</td>
<td>2</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>103</td>
<td>25</td>
<td>39</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>43</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>47</td>
<td>42</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>53</td>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>39</td>
<td>3</td>
<td>34</td>
</tr>
<tr>
<td>7</td>
<td>16</td>
<td>39</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>26</td>
<td>59</td>
<td>3</td>
<td>25</td>
</tr>
</tbody>
</table>

Solution to Problem 1: Desired quantity = B = A - 20

Solution to Problem 2: Desired quantity = B = 100

Solution to Problem 3: Desired quantity = B = 100
Obstacles to Problem Solving

- Functional fixedness: the elevator riddle:
  - The once was man named Otto.
  - Everyday, Otto rides the elevators from his apartment on the 15th floor to the lobby.
  - But when he returns, he rides the elevator to the 10th floor and then walks to the 15th.
  - But when it rains, he usually rides all the way to the 15th.
  - Why doesn’t Otto always ride all the way to the 15th floor?

Information-Processing Approach

- Newell and Simon
- Problem space
  - Initial state
  - Intermediate state(s)
  - Goal state

What Is a Problem?

- **Well-defined problem**: has a correct answer, certain procedures will lead to solution
  - A small “problem space”.

- **Ill-defined problem**: path to solution is unclear, no one “correct” answer
  - A large “problem space”.
Information-Processing Approach

- Tower of Hanoi
- Operators: actions to go from one state to another.
  - Governed by rules that specify which moves are allowed and which are not.

- Initial and goal states for the Tower of Hanoi problem.

- The operators for the Tower of Hanoi problem.
Information-Processing Approach

- Means-end analysis: reduce differences between initial and goal states
- Subgoals: create intermediate states closer to goal

- Initial steps in solving the Tower of Hanoi problem, showing how the problem can be broken down into subgoals.

The Importance of How a Problem Is Stated...

- Acrobat and reverse acrobat problem
- One small change in wording of problem
- Not just analyzing structure of problem space
- How a problem is stated can affect its difficulty
Framing

- Mutilated-checkerboard problem
- Conditions differed in how much information provided about the squares
- Easier to solve when information is provided that points toward the correct representation of the problem

Kaplan and Simon (1990)

Analogy

- Using a solution to a similar problem guides solution to new problem
- Russian marriage problem (source problem) --> mutilated-checkerboard problem (target problem)
Using Analogies to Solve a Problem

- Gick and Holyoak (1980, 1983)
  - Noticing relationship
  - Mapping correspondence between source and target
  - Applying mapping

Using Analogies to Solve a Problem

- Duncker’s Radiation Problem (1945)
  - Analogies aid problem solving
  - Often hints must be given to notice connection
    - Surface features get in the way
    - Structural features must be used
3 Steps for analogical problem solving

- **Noticing** that there is an analogous relationship between source story and target story.
- **Mapping** the correspondence between target and source stories.
- **Applying** the mapping to generate a parallel solution to the target problem

Using Analogies to Solve a Problem

- Lightbulb problem (Holyoak and Koh, 1987)
- *High surface similarities* aid analogical problem solving
- Making structural features more obvious aids analogical problem solving

Using Analogies to Solve a Problem

- **Analogical encoding**: comparing two cases that illustrate a principle (Genter & Goldwin-Meadow, 2003)
- Effective way to get participants to see structural features that aide problem solving
Using Analogies to Solve a Problem

- **Analogical paradox** (Kevin Dunbar, 2001)
  - Participants in experiments focus on surface features
  - People in the real world use structural features

Analogical Paradox

- Dunbar used **In vivo problem-solving research** to study analogical paradox
  - People are observed to determine how they solve problems in the real world
    - **Advantage**: naturalistic setting
    - **Disadvantages**: time-consuming, cannot isolate and control variables

Using Analogies to Solve a Problem

- Dunbar found that many groups use deep structural analogies frequently
  - Molecular biologists
  - Immunologists
  - Engineers
How Experts Solve Problems

- Experts solve problems in their field faster and with a higher success rate than beginners
- Experts possess more knowledge about their fields
- Knowledge is organized so it can be accessed when needed to work on a problem
  - Novice: surface features
  - Expert: deep structure

How Experts Solve Problems

Novice
The novice grouped problems 23 and 24 together because they both involve similar objects (inclined planes).

Expert
The expert grouped problems 21 and 24 together because they both involve similar physics principles (conservation of energy).

Surface features
Deep structure

(Chi et al., 1981)

How Experts Solve Problems

- Experts spend more time understanding the problem (Lesgold, 1988)
- Experts are no better than novices when given problems outside of their field
- Experts less likely to be open to new ways of looking at problems (e.g., Kuhn, 1970)
Creative Problem-Solving

- Creativity
- Innovative thinking
- Novel ideas
- New connections between existing ideas
- Divergent thinking: open-ended; large number of potential "solutions"
- Convergent thinking: one correct answer

Design fixation (Jansen & Smith, 1991)
- Fixated on what not to do as demonstrated by sample
- Fixation can inhibit problem-solving

Creative cognition: technique to train people to think creatively (Finke, 1995)
- Preinventive forms: ideas that precede creation of finished creative product
Five pirates have obtained 100 gold coins and have to divide up the loot. The pirates are all extremely intelligent, treacherous and selfish (especially the captain).

The captain always proposes a distribution of the loot. All pirates vote on the proposal, and if half the crew or more go "Aye", the loot is divided as proposed, as no pirate would be willing to take on the captain without superior force on their side.

If the captain fails to obtain support of at least half his crew (which includes himself), he faces a mutiny, and all pirates will turn against him and make him walk the plank. The pirates start over again with the next senior pirate as captain.

What is the maximum number of coins the captain can keep without risking his life?

According to the story, four prisoners are arrested for a crime, but the jail is full and the jailer has nowhere to put them. He eventually comes up with the solution of giving them a puzzle so if they succeed they can go free but if they fail they are executed.

The jailer puts three of the men sitting in a line. The fourth man is put behind a screen (or in a separate room). He gives all four men party hats (as in diagram). The jailer explains that there are two red and two blue hats. The prisoners can see the hats in front of them but not on themselves or behind. The fourth man behind the screen can't see or be seen by any other prisoner. No "communication" between the prisoners is allowed.

Then, prisoner C (see image) either calls out the color of his hat, or says he doesn't know what color it is. After this, prisoner B either calls out the color of his hat, or says he doesn't know. Similar for prisoner A. If any of the three prisoners can figure out what color hat he has on his head all four prisoners go free. The puzzle is to find how the prisoners can escape.