Traditional Names of Model Comparisons

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1 t-Test, Student's t-test

The following tests are all isomorphic to traditional t-tests in that they will reject the null hypothesis if and only if the t-test would have rejected the null hypothesis. To obtain the actual value of t, simply use the formula

$$t_{df} = \sqrt{F_{1,df}}$$

For a so-called "one-tailed t-test," report an α equal to half that actually used for the F^* and the PRE for any of the model comparisons in this section.

1.1 One-Sample t-test

The following tests whether a single mean differs from the a priori specified value of B_0.

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \epsilon_i \\ \text{MODEL C} &: Y_i = B_0 + \epsilon_i \\ H_0 &: \beta_0 = B_0 \end{aligned}$$

1.2 Two-Sample t-test (independent)

X_i is a contrast-coded predictor with $+1$ designating the members of one group and -1 the members of the other group. The following tests whether the means of the two groups are equal.

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \beta_1 X_i + \epsilon_i \\ \text{MODEL C} &: Y_i = \beta_0 + \epsilon_i \\ H_0 &: \beta_1 = 0 \end{aligned}$$

1.3 Two-Sample t-test (dependent)

There are two scores, Y_{i1} and Y_{i2} for each case. Construct $W_i = Y_{i1} - Y_{i2}$. The following tests whether the mean of the first scores for each case equal the mean of the second scores.

$$\begin{aligned} \text{MODEL A} &: W_i = \beta_0 + \epsilon_i \\ \text{MODEL C} &: W_i = 0 + \epsilon_i \\ H_0 &: \beta_0 = 0 \end{aligned}$$

2 Regression

All predictors X_{ij} are continuous.

2.1 Simple or Linear Regression

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \beta_1 X_i + \epsilon_i \\ \text{MODEL C} &: Y_i = \beta_0 + \epsilon_i \\ H_0 &: \beta_1 = 0 \end{aligned}$$

Note that this is identical to the "two-sample independent t-test." The correlation coefficient $r = \sqrt{PRE}$ with the sign of r determined by the sign of b_1.

2.2 Multiple Regression (Additive)

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \epsilon_i \\ \text{MODEL C} &: Y_i = \beta_0 + \epsilon_i \\ H_0 &: \beta_1 = 0 \textit{ and } \beta_2 = 0. \end{aligned}$$

This is easily generalized to more predictors than two. A common summary index is $R^2 = PRE$.

To test the effect of one predictor while controlling for all other predictors, use a MODEL C that omits only that predictor. For example, to test the effects of X_2 controlling for X_1 use

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \epsilon_i \\ \text{MODEL C} &: Y_i = \beta_0 + \beta_1 X_{i1} \epsilon_i \\ H_0 &: \beta_2 = 0. \end{aligned}$$

The results of this test are frequently reported as $t_{n-PA} = \sqrt{F_{1,n-PA}}$ and/or as the partial correlation coefficient $r_{Y2.1} = \sqrt{PRE}$ with the sign determined by the sign of b_2.

2.3 Quadratic or Polynomial Regression

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i1}^2 + \epsilon_i \\ \text{MODEL C} &: Y_i = \beta_0 + \beta_2 X_{i1} + \epsilon_i \\ H_0 &: \beta_2 = 0. \end{aligned}$$

2.4 Moderated Regression or Multiple Regression with Interactions

Does the simple relationship between Y and X_1 depend on the level of X_2? That is, is there an interaction between X_1 and X_2?

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1} X_{i2} + \epsilon_i \\ \text{MODEL C} &: Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \epsilon_i \\ H_0 &: \beta_3 = 0. \end{aligned}$$

Note: The product of $X_1 X_2$ represents the interaction only if both components of the product also appear separately in the model. Rewriting MODEL A as

$$\text{MODEL A} : Y_i = [\beta_0 + \beta_2 X_{i2}] + [\beta_1 + \beta_3 X_{i2}] X_{i1} + \epsilon_i$$

shows why this test is sometimes described as testing whether X_2 *moderates* the effect of X_1.

3 Analysis of Variance, ANOVA

3.1 One-Way ANOVA

A single categorical predictor variable with k levels (usually three or more levels but sometimes only two) is coded with $k-1$ orthogonal, contrast predictors. For $k = 3$ the test that the means for each level of the categorical variable are equal is given by

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \epsilon_i \\ \text{MODEL C} &: Y_i = \beta_0 + \epsilon_i \\ H_0 &: \beta_1 = 0 \textit{ and } \beta_2 = 0. \end{aligned}$$

This is the same test as for multiple regression.

3.2 Two-Way ANOVA

Two categorical predictors Row and Col. Make a set of one-way contrast codes for Row and a separate set for Col, then construct additional codes from all possible products between Row and Col codes. MODEL A contains all coded predictors. To test the main effect of Row (i.e., that the means for each Row are equal), use a MODEL C that leaves out all the Row codes. To test the main effect of Col, leave out all the Col codes. To test the interaction, whether the effect of Row depends on the level of Col (or vice-versa), leave out all the product codes.

3.3 Repeated Measures or Within-Subject ANOVA

There are multiple observations $Y_{i1}, Y_{i2}, \ldots Y_{ik}$ from each observational unit. Apply contrast codes to the Y's to form new variables W_i and then analyze those using any other technique in this list.

4 Both Continuous and Categorical Predictors

For the procedures in this section there is at least one contrast-coded predictor X_i and at least one continuous predictor Z_i. Generalizations to more than one of either are straight-forward.

4.1 Analysis of Covariance, ANCOVA

To test whether the means coded by X differ when controlling for Z, use the following models.

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \epsilon_i \\ \text{MODEL C} &: Y_i = \beta_0 + \beta_1 Z_i + \epsilon_i \\ H_0 &: \beta_1 = 0. \end{aligned}$$

4.2 Equality of Regressions

At least one continuous predictor Z_i and at least one contrast-coded predictor X_i coding the distinction between two or more regression lines. To ask

whether two regression models are required instead of one, test

$$\begin{aligned} \text{MODEL A} &: Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \beta_3 X_i Z_i + \epsilon_i \\ \text{MODEL C} &: Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \epsilon_i \\ H_0 &: \beta_3 = 0. \end{aligned}$$

This is the same as moderated regression. This is also the test for homogeneity of regression in ANCOVA. That is, it asks whether the relationship between Y and the covariate Z is the same within each level defined by X.