Judd, C.M., \& McClelland, G.H. (1989). Data Analysis: A Model Comparison Approach. HBJ.


Brief Lecture Notes for
Chapter 8.
Multiple Regression: Models with Multiple Continuous Predictors

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Why?

1. one predictor at a time is inefficient
2. may need more than 1 simultaneously
3. including more predictors in model may give more power for question of interest
use fatrate models as an example
MODELs of the form:

$$
Y_{i}=\beta_{0}+\beta_{1} X_{i 1}+\beta_{2} X_{i 2}+\cdots+\beta_{p-1} X_{i, p-1}+\varepsilon_{i}
$$

partial regression coefficient, better notation is

$$
\beta_{1.23 \ldots p-1} \quad \beta_{2.134 \ldots p-1}
$$

New Problem: Redundancy among predictors couldn't have been a problem with simple reg
e.g., JANTEMPC measured in Cent. and

JANTEMPF measured in F.---complete redundancy
more common, partial redundancy
JANTEMP and DECTEMP or
JANTEMP and JANSNOW
we will have to be alert for redundancy and learn to interpret models which involve redundancy, but that is ONLY new problem

Statistical Inference
nothing new!
can ask a lot more questions, but MODEL C/ A comparisons, PRE, and $F^{*}$ are exactly as before

## Estimation:

find $b$ 's so that

$$
\hat{Y}_{i}=b_{0}+b_{1} X_{i 1}+\cdots+b_{p-1} X_{i, p-1}
$$

SSE is minimized.
Line of simple reg generalizes to plane, see Ex 8.1, p. 154.
$b_{0}=\bar{Y}-b_{1} \bar{X}_{1}-b_{2} \bar{X}_{2}-\cdots$
If no redundancy, then just like in Chpt 6:

$$
b_{j}=\frac{\sum\left(X_{i j}-\bar{X}_{j}\right)\left(Y_{i}-\bar{Y}\right)}{\sum\left(X_{i j}-\bar{X}_{j}\right)^{2}}
$$

If redundancy, then it is a mess! We will let computer do it!

To give you an idea of the mess, here is the leastsquares estimate for one coefficient when there are two predictors.
$b_{1}=\frac{\left(\sum x_{1} y\right)\left(\sum x_{2}^{2}\right)-\left(\sum x_{2} y\right)\left(\sum x_{1} x_{2}\right)}{\left(\sum x_{1}^{2}\right)\left(\sum x_{2}^{2}\right)-\left(\sum x_{1} x_{2}\right)^{2}}$
where $\quad x_{1}=\left(X_{1}-\bar{X}_{1}\right)$, etc.
So, for two or more predictors we will be happy to let the computer do the estimation.

## Do detailed example using SAS output

Do UN \& IP example (Section 8.5) to illustrate meaning of partial regression coefficients.


