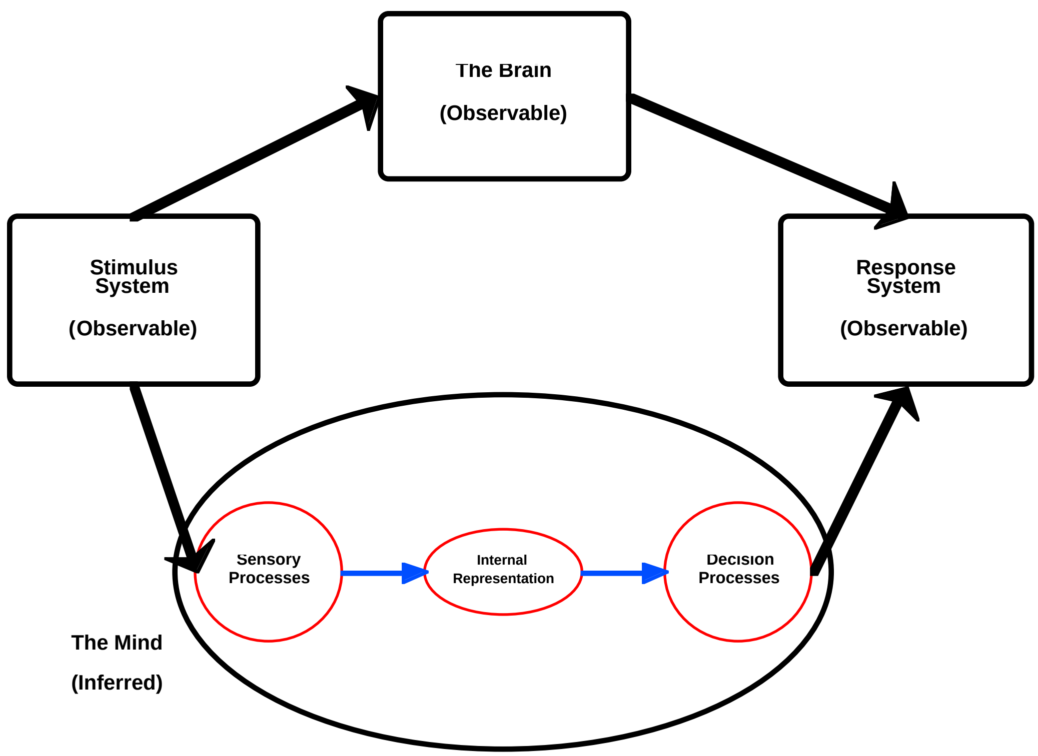
Detection Sensitivity and Response Bias

Lewis O. Harvey, Jr.
Department of Psychology
University of Colorado
Boulder, Colorado



Detection Sensitivity and Response Bias

A. Introduction

Classical psychophysical methods had as their goal the determination of a stimulus threshold. Types of thresholds include detection, discrimination, recognition, and identification. What is a threshold? The concept of threshold actually has two meanings: One empirical and one theoretical. Empirically speaking, a threshold is the stimulus level that will allow the observer to perform a task (detection, discrimination, recognition, or identification) at some criterion level of performance (75% correct, for example). Theoretically speaking, a threshold is property of a model or theory of detection or descrimination behavior. All models of detection and discrimination have at least two psychological components or processes: the sensory process (which transforms physical stimulation into internal sensations) and a decision process (which decides on responses based on the output of the sensory process (Krantz, 1969) as illustrated in Figure 1.

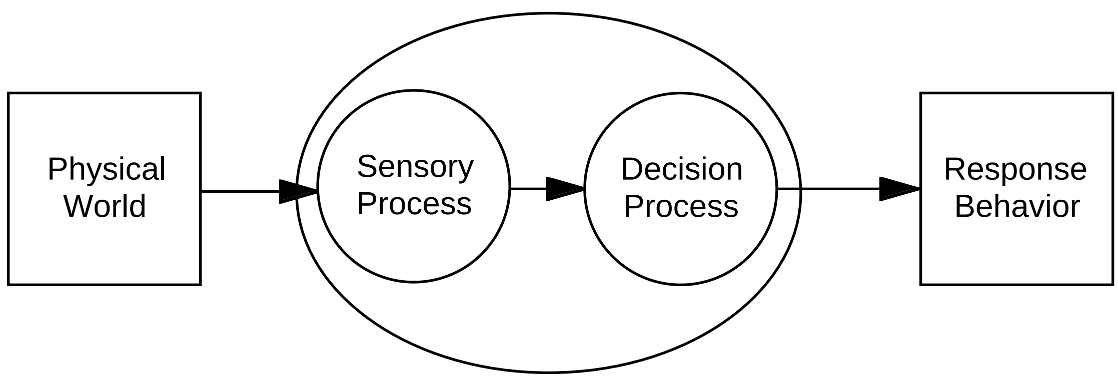


Figure 1: Detection is based on two internal processes

The classical concept of a detection threshold, as represented in the high threshold model (HTM) of detection, is a property of the sensory process. It is a stimulus level below which the stimulus has no effect (as if the stimulus were not there) and above which the stimulus causes the sensory process to generate an output. The classical psychophysical methods (the method of limits, the method of adjustment, and the method of constant stimuli) developed by Gustav Theodor Fechner (1860) were designed to infer the stimulus value corresponding to the theoretical threshold from the observed detection performance data. In this sense, the stimulus threshold is the stimulus energy that exceeds the theoretical threshold with a probability of 0.5. Until the 1950s the high threshold model of detection dominated our conceptualization of the detection process and provided the theoretical basis for the psychophysical measurement of thresholds.

In the 1950s a major theoretical advance was made by combining detection theory with statistical decision theory. Detection performance, within this new framework, is also based on a sensory process and a decision process. The sensory process transforms the physical stimulus energy into some sort of internal representation and the decision process decides what response to make based on this internal representation. The response can be a simple yes or no ("yes, the stimulus was present" or "no, the stimulus was not present") or a more elaborate response, such as a rating of the confidence that the signal was present. Each of the two processes is characterized by at least one parameter: The sensory process by a sensitivity parameter and the decision process by a response criterion or response bias parameter.

It was further realized that estimates of thresholds made by the three classical psychophysical methods confounded the sensitivity of the sensory process with the response criterion of the decision process. In order to measure these two separate characteristics, one needs to measure two aspects of detection performance. Not only must one measure the conditional probability that the observer says "yes" when a stimulus is present (the hit rate, or HR) but also one must measure the conditional probability that the observer says "yes" when a stimulus is not present (the false alarm rate, or FAR). These conditional probabilities are shown in Table 1.Within the framework of a model, these two performance measures, HR and FAR, may be used to estimate detection sensitivity and decision criterion. The specific way in which the HR and FAR are used to compute detection sensitivity and response criterion depends on the specific model one adopts for the sensory process and for the decision process. Some of these different models and how to distinguish among them are discussed in a classic paper by David Krantz (1969). The major two competing models discussed below are the high threshold model and the signal detection theory.

Table 1: Conditional probabilities in the simple detection paradigm.

	"Yes"	"No"
Signal Present	Hit Rate (HR)	Miss Rate (MR)
Signal Absent	False Alarm Rate (FAR)	Correct Rejection Rate (CRR)

B. High Threshold Model of Detection

The high threshold model (HTM) of detection assumes that the sensory process contains a sensory threshold. When a stimulus is above threshold, the sensory process generates an output and the decision process says “yes.” On trials when the stimulus is below threshold and the sensory process therefore does not generate an output, the decision process decides to say “yes” anyway, a guess. In the high threshold model (HTM) the measures of sensory process sensitivity and decision process guessing rate are computed from the observed hit rate (HR) and false alarm rate (FAR):

$$p = \frac{HR - FAR}{1 - FAR} \qquad \text{Sensitivity of the Sensory Process} \qquad (1)$$

$$g = FAR \qquad \text{Guessing Rate of the Decision Process} \qquad (2)$$

where p is the probability that the stimulus will exceed the threshold of the sensory process and g is the guessing rate of the decision process (guessing rate is the decision criterion of the high threshold model). Equation 1 is also called the correction-for-guessing formula.

The High Threshold Model is not valid. Extensive research testing the validity of the high threshold model has lead to its rejection: It is not an adequate description of the detection process and therefore Equations 1 and 2 do not succeed in separating the effects of sensitivity and response bias (Green & Swets, 1966/1974; Krantz, 1969; Macmillan & Creelman, 1991; McNicol, 1972; Swets, 1961, 1986a, 1986b, 1996; Swets, Tanner, & Birdsall, 1961). The reasons for rejecting the high threshold model are discussed next.

The Receiver Operating Characteristic: One important characteristic of any detection model is the predictted relationship between the hit rate and the false alarm rate as the observer changes decision strategy. The plot of HR vs. FAR is called an ROC (receiver operating characteristic). By algebraic rearrangement of Equation 1, the high threshold model of detection predicts a linear relationship between HR and FAR:

$$HR = p + (1 - p) \cdot FAR \qquad \text{Receiver Operating Characteristic} \qquad (3)$$

where p is the sensitivity parameter of the high threshold sensory process. This predicted ROC is shown in Figure 2. When one measures the hit rate and false alarm rate in a detection experiment using different degrees of response bias, a bowed-shaped ROC (shown by the filled circles in Figure 2) is obtained. This bowed-shaped ROC is obviously quite different from the straight line relationship predicted by the high threshold model and is one of the bases for rejecting that model.

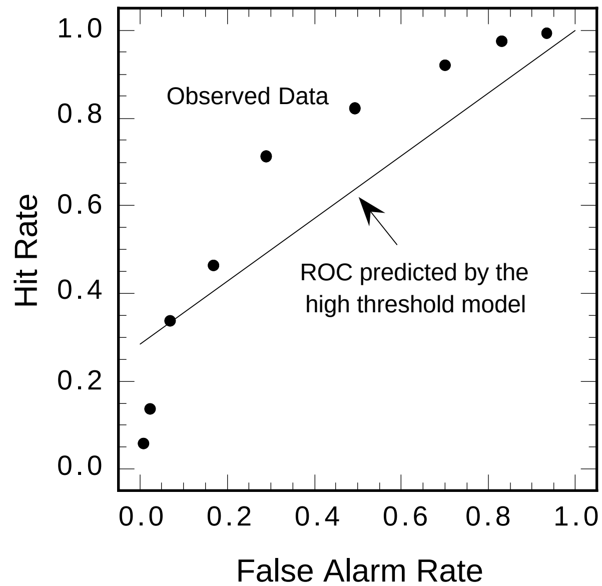


Figure 2: The Receiver Operating Characteristic (ROC) predicted by the high threshold model of detection compared with typical data.

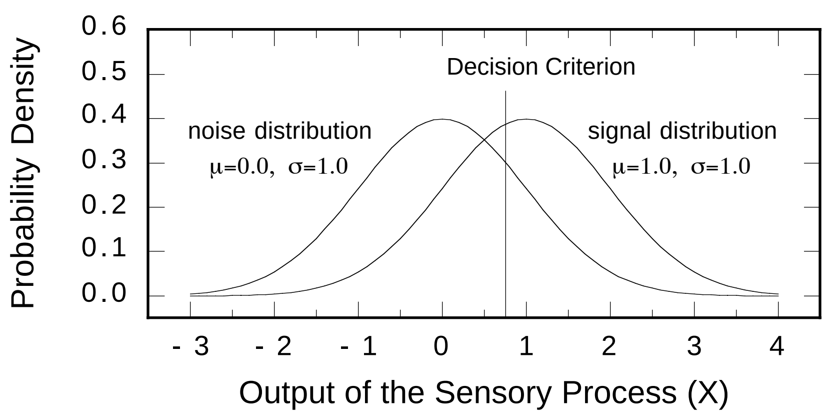


Figure 3: Gaussian probability functions of getting a specific output from the sensory process without and with a signal present. The vertical line is the decision criterion, X_c. Outputs higher than X_c lead to a *yes* response; those lower or equal to X_c lead to a *no* response.

C. Signal Detection Theory

A widely accepted alternative to the high threshold model was developed in the 1950s and is called signal detection theory (Harvey, 1992). In this model the sensory process has no sensory threshold (Swets, 1961; Swets et al., 1961; Tanner & Swets, 1954). The sensory process is assumed to have a continuous output based on random Gaussian noise and that when a signal is present, the signal combines with that noise. By assumption, the noise distribution has a mean, μ_n, of 0.0 and a standard deviation, σ_n, of 1.0. The mean of the signal-plus-noise distribution, μ_s, and its standard deviation, σ_s, depend upon the sensitivity of the sensory process and the strength of the signal. These two Gaussian probability distributions are seen in Figure 3. Model based on other probability distributions are also possible (Egan, 1975).

Measures of the sensitivity of the sensory process are based on the difference between the mean output under no signal condition and that under signal condition. When the standard deviations of the two distributions are equal ($\sigma_n = \sigma_s = 1$) sensitivity may be represented by d' (pronounced 'd-prime'):

$$d' = \frac{(\mu_s - \mu_n)}{\sigma_n} \qquad \text{Equal-Variance Sensitivity} \qquad (4)$$

In the more general case, when $\sigma_n \neq \sigma_s$ (Simpson & Fitter, 1973; Swets, 1986a, 1986b), the appropriate measure of sensitivity is d_a ("d-sub-a"):

$$d_a = \frac{(\mu_s - \mu_n)}{\sqrt{\dfrac{\sigma_s^2 + \sigma_n^2}{2}}} \qquad \text{Unequal-Variance Sensitivity} \qquad (5)$$

Note that in the case when $\sigma_n = \sigma_s$ (equal-variance model), $d_a = d'$.

The decision process is assumed to use one or more decision criteria. The output of the sensory process on an experimental trial is compared to the decision criteria to determine which response to give. In the case of one decision criterion, for example, if the output of the sensory process equals or exceeds the decision criterion, the observer says "yes, the signal was present." If the output of the sensory process is less than this criterion, the observer says "no, the signal was not present."

Receiver Operating Characteristic: The ROC predicted by the signal detection model is shown in the left panel of Figure 4 along with the observed data from Figure 2. The signal detection prediction is in accord with the observed data. The data shown in Figure 4 are fit by a model having $\mu_s = 1$, $\sigma_s = 1$, with a sensitivity of $d_a = 1$. The fitting of the model to the data was done using a maximum-likelihood algorithm: the program, Rscore+, is available from the author. The ROC predicted by the signal detection theory model is anchored at the (0,0) and (1,1) points on the graph. Different values of μ_s generate a different ROC. For $\mu_s = 0$, the ROC is the positive diagonal extending from (0,0) to (1,1). For μ_s greater than zero, the ROC's are bowed. As μ_s increases so does the bowing of the corresponding ROC as may be seen in the right panel of Figure 4 where the ROC's of four different values of μ_s are plotted.

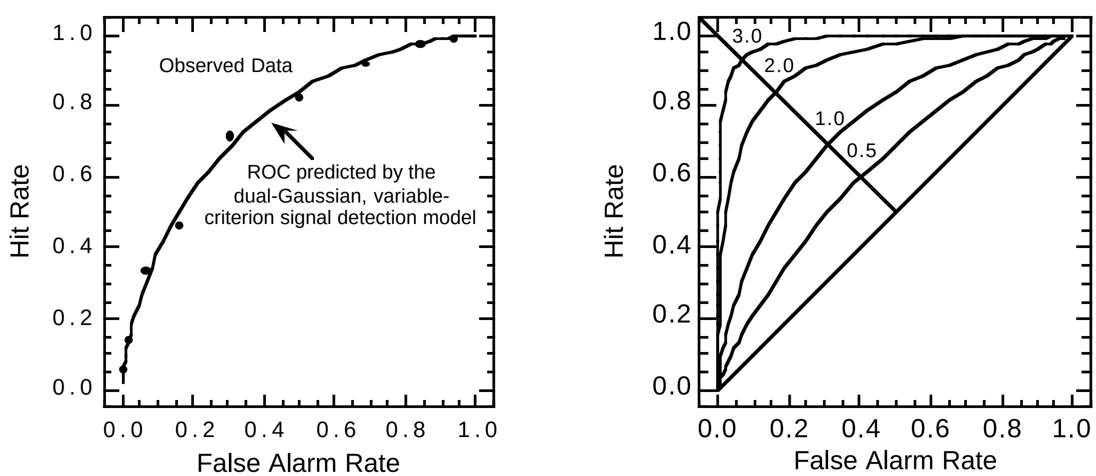


Figure 4: Left panel: Receiver Operating Characteristic (ROC) predicted by Signal Detection Theory compared with typical data. Right Panel: ROC's of four different models. The ROC becomes more bowed as the mean signal strength increases.

The equation for the signal detection theory ROC is that of a straight line if the HR and FAR are expressed as z-scores of the unit, normal Gaussian probability distribution:

$$z(HR) = \mu_s \cdot \frac{\sigma_n}{\sigma_s} + \frac{\sigma_n}{\sigma_s} \cdot z(FAR) \qquad \text{Signal Detection Theory ROC} \qquad (6a)$$

where $z(HR)$ and $z(FAR)$ are the z-score transforms of the HR and FAR probabilities, based on the Gaussian normal distribution. That Equation 6a is linear is easily seen if we let $a = \mu_s \cdot (\sigma_n / \sigma_s)$, and let $b = \sigma_n / \sigma_s$ which gives:

$$z(HR) = a + b \cdot z(FAR) \qquad \text{Signal Detection Theory ROC} \qquad (6b)$$

The values of the y-intercept *a* and the slope *b* of this ROC are directly related to the mean and standard deviation of the signal plus noise distribution:

$$\mu_s = \frac{a}{b} + \mu_n \qquad \text{Mean of Signal plus Noise} \qquad (7)$$

$$\sigma_s = \frac{\sigma_n}{b} = \frac{1}{b} \qquad \text{Standard Deviation of Signal plus Noise} \qquad (8)$$

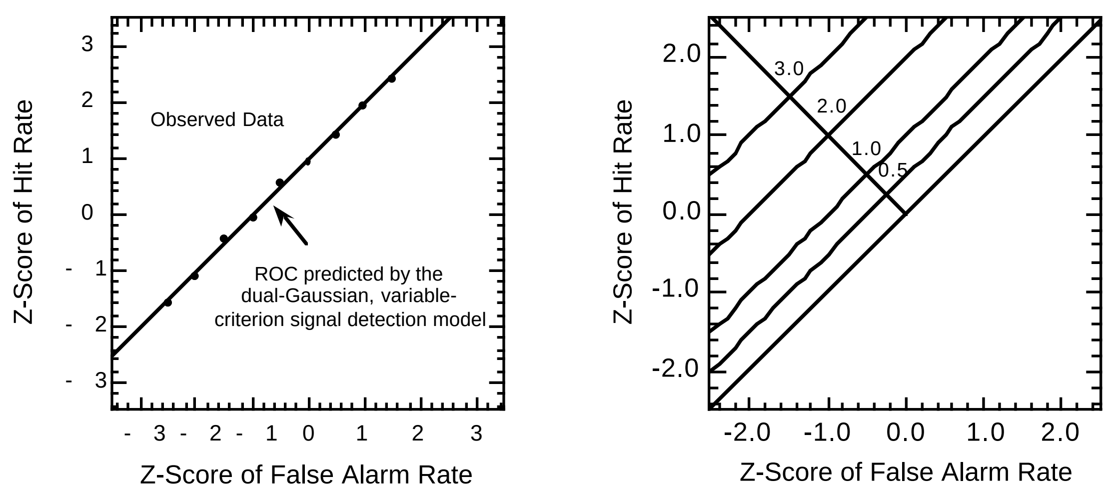


Figure 5: Left panel: Receiver Operating Characteristic (ROC) predicted by Signal Detection Theory compared with typical data. Right panel: ROC's of four different models. The ROC is farther from the diagonal as the mean signal strength increases.

Equation 6 predicts that when the hit rate and the false alarm rate are transformed from probabilities into z-scores of the unit, normal Gaussian probability distribution, the ROC will be a straight line. The z-score transformation from probability is made using tables that are in every statistics textbook. Some scientific calculators can compute the transformation. Short computer subroutines based on published algorithms are also available

(Press, Teukolsky, Vetterling, & Flannery, 1992; Zelen & Severo, 1964). These routines are built into many spread sheet and graphing programs. The ROC predicted by signal detection theory is shown in the left panel of Figure 5, along with the observed data from the previous figures. The actual data are fit quite well by a straight line. The right panel of Figure 5 shows the four ROCs from Figure 4. As the mean of the signal distribution moves farther from the noise distribution the z-score ROC moves farther away from the positive diagonal.

Sensitivity of the Sensory Process: Sensitivity may be computed either from the parameters *a* and *b* of the linear ROC equation (after they have been computed from the data) or from the observed HR and FAR pairs of conditional probability:

$$d_a = \sqrt{\frac{2}{1+b^2}} \cdot a \qquad \text{(9a)}$$

$$d_a = \sqrt{\frac{2}{1+b^2}} \cdot \left(z(HR) - b \cdot z(FAR)\right) \qquad \text{(General Model)} \qquad \text{(9b)}$$

In the equal-variance model, Equation 9b reduces to the simple form:

$$d_a = d' = z(HR) - z(FAR) \qquad \text{(Equal-Variance Model)} \qquad \text{(9c)}$$

Criteria of the Decision Process: The decision process decision criterion or criteria may be expressed in terms of a critical output of the sensory process:

$$X_c = -z(FAR) \qquad \text{Decision Criterion} \qquad \text{(10)}$$

The decision process decision criterion may also be expressed in terms of the likelihood ratio that the signal was present, given a sensory process output of *x*:

$$\beta = \frac{\dfrac{1}{\sigma_s\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu_s}{\sigma_s}\right)^2}}{\dfrac{1}{\sigma_n\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu_n}{\sigma_n}\right)^2}} \qquad \text{Likelihood Ratio Decision Criterion} \qquad \text{(11)}$$

A way of expressing response bias is given in Equation 12 (Macmillan & Creelman, 1991):

$$c = -\frac{Z(HR) + Z(FAR)}{2} \qquad \text{Response Bias} \qquad (12)$$

Sensitivity is generally a relatively stable property of the sensory process, but the decision criterion used by an observer can vary widely from task to task and from time to time. The decision criterion used is influenced by three factors: The instructions to the observer; the relative frequency of signal trial and no-signal trails (the *a priori* probabilities); and the payoff matrix, the relative cost of making the two types of errors (False Alarms and Misses) and the relative benefit of making the two types of correct responses (Hits and Correct Rejections). These three factors can cause the observer to use quite different decision criteria at different times and if the proper index of sensitivity is not used, changes in decision criterion will be incorrectly interpreted as changes in sensitivity.

D. More Reasons to Reject the High Threshold Model

Figure 6 shows the high threshold sensitivity index *p* for different values of decision criteria, for an observer having constant sensitivity. The decision criterion is expressed in terms of the HTM by *g* and the SDT by X_c The detection sensitivity *p* calculated from Equation 1, is not constant, but changes as a function of decision criterion.

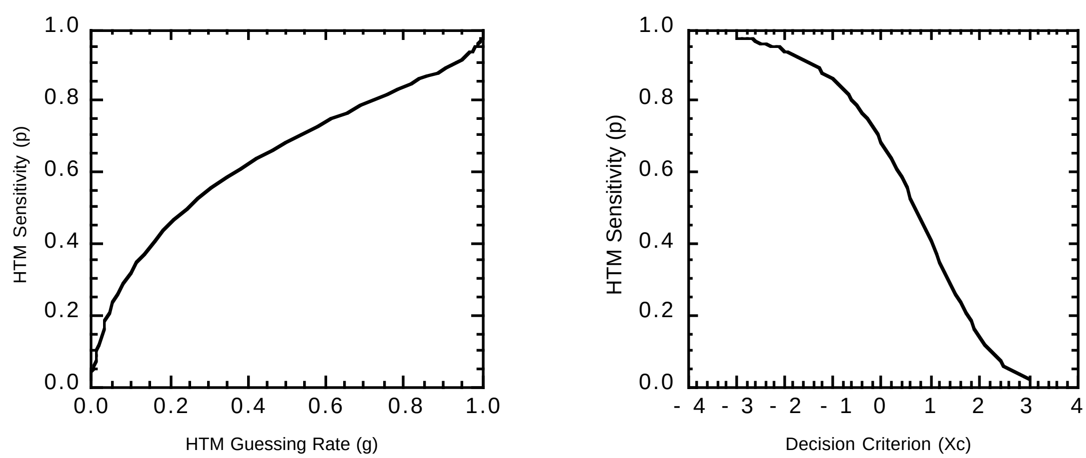


Figure 6: Sensitivity, p, of HTM sensory process computed from HR and FAR in the "yes-no" paradigm as a function of the guessing rate g (left panel) or the decision criterion, X_c, (right panel). The High Threshold Model predicts that p should remain constant.

Another popular index of sensitivity is overall percent correct (hit rate and correct rejection rate combined). In Figure 7 percent correct is plotted as a function of decision criterion. One sees in Figure 7 that percent correct also does not remain constant with changes in decision criterion. This failure to remain constant is another reason for rejecting the high threshold model.

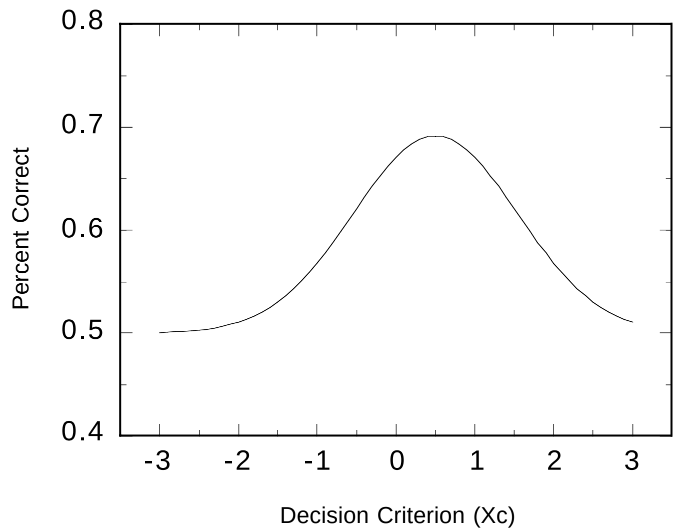


Figure 7: Overall percent correct in a "yes-no" experiment for different values of the decision criterion.

E. Two-Alternative, Forced-Choice Detection Paradigm:

Forced-choice, especially two-alternative, forced-choice (2AFC), is a widely-used paradigm which is an alternative to the single-interval “yes-no” paradigm discussed above. Because only one performance index, percent correct, is obtained from this paradigm, it is not possible to calculate both a detection sensitivity index and a response criterion index. Detection performance in the 2AFC paradigm is equivalent to an observer using an unbiased decision criterion, and the percent correct performance can be predicted from signal detection theory. Percent correct in a 2AFC detection experiment corresponds to the area under the ROC, A_z, obtained when the same stimulus is used in the yes-no signal detection paradigm. Calculation of d_a from the 2AFC percent correct is straight forward:

$$d_a = \sqrt{2} \cdot z(pc) \qquad \text{(Two-Alternative, Forced-Choice)} \qquad (12)$$

where $z(pc)$ is the z-score transform of the 2AFC percent correct (Egan, 1975; Green & Swets, 1966/1974; Macmillan & Creelman, 1991; Simpson & Fitter, 1973). The area under the ROC for $d_a = 1.0$, illustrated in Figure 2 and the left panel in Figure 4, is 0.76 (the maximum area of the whole graph is 1.0). By rearranging Equation 13, the area under the ROC may be computed from d_a by:

$$A_z = z^{-1}\left(\frac{d_a}{\sqrt{2}}\right) \qquad \text{(Two-Alternative, Forced-Choice)} \qquad (13)$$

where $z^{-1}(\ )$ is the inverse z-score probability transform that converts a z-score back into a probability.

F. Summary

The classical psychophysical methods of limits, of adjustment, and of constant stimuli, provide procedures for estimating sensory thresholds. These methods, however, are not able to properly separate the independent factors of sensitivity and decision criterion. Furthermore, there is no evidence to support the existence of sensory thresholds, at least in the form these methods were designed to measure.

Today there are two methods that allow one to measure an observer's detection sensitivity relatively uninfluenced by changes in decision criteria. The first method is based on signal detection theory, and requires that there be two types of detection trials: Some containing the signal and some containing no signal. Both detection sensitivity and response criterion may be calculated from the hit rates and false alarm rates resulting from the performance in these experiments. The second method is the forced-choice paradigm, which forces all observers to adopt the same decision criterion. Either of these methods may be used to measure psychometric functions. The "threshold" stimulus level corresponds to the stimulus giving rise to a certain level of detection performance. A d_a of 1.0 or a 2AFC detection of 0.75 are often used to define threshold, but other values may be chosen as long as they are made explicit.

One advantage of a detection sensitivity measure which is uncontaminated by decision criterion is that this measure may be used to predict actual performance in a detection task under a wide variety of different decision criteria. It is risky and without justification to assume that the decision criterion that the observer adopts in the laboratory is the same when performing a real-world detection task.

A second advantage is that variability in measured sensitivity is reduced because the variability due to changes in decision criteria is removed. A comparison of contrast sensitivity functions measured by means of the method of adjustment (contaminated by decision criterion) and the two-alternative, forced-choice method (not contaminated by decision criterion) was reported by Higgins, Jaffe, Coletta, Caruso, and de Monasterio (1984). The variability of the 2AFC measurements is less than one half those made with the method of adjustment. This reduction of measurement variability will increase the reliability of the threshold measures and increase its predictive validity.

The material above concerns the behavior of an ideal observer. Although the benefits of using a bias-free measure of sensitivity are clear, there may be circumstances where less than ideal psychophysical procedures must be employed. Factors such as testing time, ease of administration, ease of scoring, and cost must be carefully considered in relationship to the desired reliability, accuracy, and ultimate use to which the measurements will be put. Finally, it must be recognized that no psychophysical method is perfect. Observers may make decisions in irrational ways. Some observers may try to fake a loss of sensory capacity. Care must be taken, regardless of the psychophysical method used to measure capacity, to detect such malingering. But a properly administered, conceptually rigorous psychophysical procedure will insure the maximum predictive validity of the measured sensory capacity.

References

Egan, J. P. (1975). *Signal Detection Theory and ROC Analysis*. New York: Academic Press.

Fechner, G. T. (1860). *Elemente der Psychophysik*. Leipzig: Breitkopf and Härtel.

Green, D. M., & Swets, J. A. (1966/1974). *Signal detection theory and psychophysics* (A reprint, with corrections of the original 1966 ed.). Huntington, New York: Robert E. Krieger Publishing Co.

Harvey, L. O., Jr. (1992). The critical operating characteristic and the evaluation of expert judgment. *Organizational Behavior & Human Decision Processes, 53*(2), 229-251.

Higgins, K. E., Jaffe, M. J., Coletta, N. J., Caruso, R. C., & de Monasterio, F. M. (1984). Spatial contrast sensitivity: Importance of controlling the patient’s visibility criterion. *Archives of Ophthalmology, 102*, 1035-1041.

Krantz, D. H. (1969). Threshold theories of signal detection. *Psychological Review, 76*(3), 308-324.

Macmillan, N. A., & Creelman, C. D. (1991). *Detection theory: A user's guide*. Cambridge: Cambridge University Press.

McNicol, D. (1972). *A primer of signal detection theory*. London: George Allen & Unwin.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (1992). *Numerical Recipes in C: The Art of Scientific Computing* (2nd ed.). New York: Cambridge University Press.

Simpson, A. J., & Fitter, M. J. (1973). What is the best index of detectability? *Psychological Bulletin, 80*(6), 481-488.

Swets, J. A. (1961). Is there a sensory threshold? *Science*, *134*, 168-177.

Swets, J. A. (1986a). Form of empirical ROC's in discrimination and diagnostic tasks: Implications for theory and measurement of performance. *Psychological Bulletin, 99*(2), 181-198.

Swets, J. A. (1986b). Indices of discrimination or diagnostic accuracy: Their ROC's and implied models. *Psychological Bulletin, 99*(1), 100-117.

Swets, J. A. (1996). *Signal Detection Theory and ROC Analysis in Psychology and Diagnostics: Collected Papers*. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Swets, J. A., Tanner, W. P., Jr., & Birdsall, T. G. (1961). Decision processes in perception. *Psychological Review, 68*(5), 301-340.

Tanner, W. P., Jr., & Swets, J. A. (1954). A decision-making theory of visual detection. *Psychological Review*, *61*(6), 401-409.

Zelen, M., & Severo, N. C. (1964). Probability functions. In M. Abramowitz & I. A. Stegun (Eds.), *Handbook of Mathematical Functions* (pp. 925-995). New York: Dover.