A Novel Perspective On Cognitive Control: Dynamic Inference Under Uncertainty

Jeremy Reynolds
University of Denver

Michael Mozer
University of Colorado, Boulder
Wisconsin Card Sorting Task
Wisconsin Card Sorting Task

Sort cards into four piles by color, symbol, or numerosity
Wisconsin Card Sorting Task

Sort cards into four piles by color, symbol, or numerosity.

Participants not told the sorting dimension; must identify by trial and error.

Dimension switches occasionally.
In order to perform task, participant must *infer* the task set.

- **Color symbol numerosity**
  - Color: red
  - Symbol: star
  - Numerosity: 3
In order to perform task, participant must *infer* the task set.
In order to perform task, participant must *infer* the task set.
In order to perform task, participant must *infer* the task set.

\[ \Rightarrow \text{uncertainty in representation of task set} \]

\[
\begin{align*}
\text{P(color)} &= 0 \\
\text{P(symbol)} &= 0.5 \\
\text{P(numerosity)} &= 0.5
\end{align*}
\]
<table>
<thead>
<tr>
<th>Task Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional Conceptualization</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
## Task Set

<table>
<thead>
<tr>
<th>Maintenance and Updating</th>
<th>Traditional Conceptualization</th>
<th>Our Conceptualization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Task Set

<table>
<thead>
<tr>
<th>Maintenance and Updating</th>
<th>Traditional Conceptualization</th>
<th>Our Conceptualization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>If-then rules that use working memory as a passive receptacle for task set</td>
<td>Inference over a stimulus-response-feedback sequence</td>
</tr>
</tbody>
</table>
## Task Set

<table>
<thead>
<tr>
<th>Maintenance and Updating</th>
<th>Traditional Conceptualization</th>
<th>Our Conceptualization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>If-then rules that use working memory as a passive receptacle for task set</td>
<td>Inference over a stimulus-response-feedback sequence</td>
</tr>
<tr>
<td>Representation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maintenance and Updating</td>
<td>Traditional Conceptualization</td>
<td>Our Conceptualization</td>
</tr>
<tr>
<td>--------------------------</td>
<td>------------------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td></td>
<td>If-then rules that use working memory as a passive receptacle for task set</td>
<td>Inference over a stimulus-response-feedback sequence</td>
</tr>
<tr>
<td>Representation</td>
<td>All-or-none, symbolic</td>
<td>Graded, probabilistic</td>
</tr>
</tbody>
</table>
Probabilistic Models

Often framed as *generative models*
Probabilistic Models

Often framed as *generative models*

Generative Model of ICS Faculty

<table>
<thead>
<tr>
<th>Gender</th>
<th>Age</th>
<th>Hair</th>
<th>Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>male</td>
<td>55</td>
<td>grey</td>
<td>comp sci</td>
</tr>
<tr>
<td>female</td>
<td>35</td>
<td>med. length</td>
<td>linguistics</td>
</tr>
<tr>
<td>male</td>
<td>45</td>
<td>grungy beard</td>
<td>comp sci</td>
</tr>
<tr>
<td>male</td>
<td>55</td>
<td>grey</td>
<td>psychology</td>
</tr>
<tr>
<td>female</td>
<td>40</td>
<td>long</td>
<td>comp sci</td>
</tr>
</tbody>
</table>
Generative Model of Control Tasks

task set

T
Generative Model of Control Tasks

task set

T

color, symbol, number
Generative Model of Control Tasks

task set

T

color, symbol, number

S-R mapping

M
Generative Model of Control Tasks

task set

S-R mapping

T

color, symbol, number

M

pile 3

pile 1
### Generative Model of Control Tasks

**Task Set**
- \( T \)
  - color, symbol, number

**S-R Mapping**
- \( M \)
  - \( \star \rightarrow \text{pile}_3 \)
  - \( \circ \rightarrow \text{pile}_1 \)
  - 4 items \( \rightarrow \text{pile}_4 \)

<table>
<thead>
<tr>
<th>( T )</th>
<th>color</th>
<th>symbol</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>color</td>
<td>.00</td>
<td>.25</td>
<td>.00</td>
</tr>
<tr>
<td>symbol</td>
<td>.25</td>
<td>.00</td>
<td>.00</td>
</tr>
<tr>
<td>number</td>
<td>.00</td>
<td>.00</td>
<td>.25</td>
</tr>
</tbody>
</table>

**P\( (M|T) \)**
Generative Model of Control Tasks

- Task set
- S-R mapping
- Stimulus
- Response
Generative Model of Control Tasks

task set

S-R mapping

category

stimulus

response
Generative Model of Control Tasks

task set

S-R mapping
category

stimulus
response
Generative Model Can Be Used To Infer Responses

What if task set is uncertain?

S-R mapping

T

M

C

S

R

P(R | S)

What if task set is uncertain?
Model Response Affected By Uncertainty

Key to explaining behavioral phenomena

Where does task set come from?
Where Does Task Set Come From?

*Inferred* from recent trial history

... just as you might looking over someone’s shoulder

```
<table>
<thead>
<tr>
<th>color</th>
<th>symbol</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>blue</td>
<td>●</td>
<td>1</td>
</tr>
<tr>
<td>purple</td>
<td>■</td>
<td>2</td>
</tr>
<tr>
<td>red</td>
<td>★</td>
<td>3</td>
</tr>
<tr>
<td>green</td>
<td>▲</td>
<td>4</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>color</th>
<th>symbol</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>blue</td>
<td>●</td>
<td>1</td>
</tr>
<tr>
<td>purple</td>
<td>■</td>
<td>2</td>
</tr>
<tr>
<td>red</td>
<td>★</td>
<td>3</td>
</tr>
<tr>
<td>green</td>
<td>▲</td>
<td>4</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>color</th>
<th>symbol</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>blue</td>
<td>●</td>
<td>1</td>
</tr>
<tr>
<td>purple</td>
<td>■</td>
<td>2</td>
</tr>
<tr>
<td>red</td>
<td>★</td>
<td>3</td>
</tr>
<tr>
<td>green</td>
<td>▲</td>
<td>4</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>color</th>
<th>symbol</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>blue</td>
<td>●</td>
<td>1</td>
</tr>
<tr>
<td>purple</td>
<td>■</td>
<td>2</td>
</tr>
<tr>
<td>red</td>
<td>★</td>
<td>3</td>
</tr>
<tr>
<td>green</td>
<td>▲</td>
<td>4</td>
</tr>
</tbody>
</table>
```
Inferring Task Set From Recent Trial History

trial index

\[ T_{i-2} \rightarrow M_{i-2} \rightarrow C_{i-2} \rightarrow S_{i-2} \]

\[ T_{i-1} \rightarrow M_{i-1} \rightarrow C_{i-1} \rightarrow S_{i-1} \]

\[ T_i \rightarrow M_i \rightarrow C_i \rightarrow S_i \]

\[ \text{trial index} \]
Inferring Task Set

\[ P(T_i | T_{i-1}) \]

<table>
<thead>
<tr>
<th></th>
<th>color</th>
<th>symbol</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{i-1} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>color</td>
<td>.90</td>
<td>.05</td>
<td>.05</td>
</tr>
<tr>
<td>symbol</td>
<td>.05</td>
<td>.90</td>
<td>.05</td>
</tr>
<tr>
<td>number</td>
<td>.05</td>
<td>.05</td>
<td>.90</td>
</tr>
</tbody>
</table>

\( T_i \)

\( T_{i-2} \) \( T_{i-1} \) \( T_i \)

\( M_{i-2} \) \( M_{i-1} \) \( M_i \)

\( C_{i-2} \) \( C_{i-1} \) \( C_i \)

\( S_{i-2} \) \( S_{i-1} \) \( S_i \)

\( R_{i-2} \) \( R_{i-1} \) \( R_i \)
Inferring Task Set

\[ P(T_i \mid S_{i-1}, R_{i-1}, S_{i-2}, R_{i-2}, ...) \]
Inferring Task Set

\[ P(T_i \mid S_{i-1}, \hat{R}_{i-1}, F_{i-1}, S_{i-2}, \hat{R}_{i-2}, F_{i-2}, \ldots) \]

stimulus  
actual response  
feedback
Predicting Human Behavior From The Model
Predicting Human Behavior From The Model

1. Given recent stimulus-response-feedback history, compute posterior over task set.
Predicting Human Behavior From The Model

1. Given recent stimulus-response-feedback history, compute posterior over task set

2. Given task set posterior, current stimulus, and desire for positive feedback, determine the response.
Free Parameters of Model
Free Parameters of Model

• $\lambda$, subjective belief that task set is unchanged from trial to trial corresponds to working memory strength incorporated into $P(T_i|T_{i-1})$

<table>
<thead>
<tr>
<th>$T_{i-1}$</th>
<th>color</th>
<th>symbol</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>color</td>
<td>$\lambda$</td>
<td>$(1-\lambda)/2$</td>
<td>$(1-\lambda)/2$</td>
</tr>
<tr>
<td>symbol</td>
<td>$(1-\lambda)/2$</td>
<td>$\lambda$</td>
<td>$(1-\lambda)/2$</td>
</tr>
<tr>
<td>number</td>
<td>$(1-\lambda)/2$</td>
<td>$(1-\lambda)/2$</td>
<td>$\lambda$</td>
</tr>
</tbody>
</table>
Free Parameters of Model

- \( \lambda \), subjective belief that task set is unchanged from trial to trial corresponds to working memory strength incorporated into \( P(T_i | T_{i-1}) \)

- \( \varepsilon \), subjective task familiarity incorporated into \( P(M_i | T_i) \)

\[
\begin{array}{cccc}
\text{T} & \text{color} & \text{symbol} & \text{number} \\
\hline
\text{M} & \starrightarrow 3 & (1-\varepsilon)/N & .25\varepsilon & (1-\varepsilon)/N \\
& \circrightarrow 1 & .25\varepsilon & (1-\varepsilon)/N & (1-\varepsilon)/N \\
4 \text{ items} \rightarrow 4 & (1-\varepsilon)/N & (1-\varepsilon)/N & .25\varepsilon \\
\cdots & \cdots & \cdots & \cdots \\
\end{array}
\]
Simulation Results
Simulation Results

Also fit

- data pattern when dimensional cues are given.
- schizophrenic/patient data via pathological model parameters
Key Principles Of Our Approach

• task set has intrinsic uncertainty

• task set updated via probabilistic inference, dependent on stimulus-response-feedback sequence

• response selection via probabilistic inference

These same principles are useful for characterizing more typical control tasks, in which participants are given explicit instructions as to the task they are to perform.
Stroop Task

Read ink color

GREEN    incongruent
BLUE     congruent

Modeling Stroop

Two task sets: color naming, word reading
Word reading better learned
Bayesian accumulator model to read out reaction times
RT depends on task set uncertainty and task set familiarity
Adds two additional free parameters (4 total)
Stroop Results I

Color Naming

- Human
- Model

Word Reading

- Congruent
- Neutral
- Incongruent

response latency
Stroop Results II

Mostly Congruent

Mostly Neutral

Mostly Incongruent

response latency

congruent neutral incongruent

human model human model human model
Virtues Of Our Approach

A novel conceptualization of cognitive control

uncertainty of task set is key to understanding control

task-set inference is a convenient way of thinking about updating and maintenance of working memory

no explicit control mechanisms: we’ve really done away with homunculus
Virtues Of Our Approach

A novel conceptualization of cognitive control

Model is strongly constrained

• Bayesian inference
• task definition and statistics
• trial sequence
Virtues Of Our Approach

A novel conceptualization of cognitive control

Model is strongly constrained

Extremely compact model

- 2-5 free parameters
- parameters are psychologically and clinically relevant
Virtues Of Our Approach

A novel conceptualization of cognitive control

Model is strongly constrained

Extremely compact model

Able to make strong predictions from a wide range of tasks

• Koechlin, Ody, & Kouneiher (2003)
• AX-CPT
• task switching
• Novel predictions we are testing experimentally
Virtues Of Our Approach

A novel conceptualization of cognitive control

Model is strongly constrained

Extremely compact model

Able to make strong predictions from a wide range of tasks

Probabilities may be the way to coherently think about the interface between symbolic and subsymbolic computation

any symbolic model can be mapped to a probabilistic representation (with probabilities = 1 or 0)

any distributed neural model can be mapped to a probabilistic representation (with 0 < probability < 1)
Neural Interpretation

anterior lateral PFC

posterior lateral PFC

posterior visual areas

premotor cortex
basal ganglia
Is There Anything To This Mapping?

Koechlin, Ody, & Kouneiher (2003)

- **premotor cortex**
- **posterior lateral PFC**
- **anterior lateral PFC**

### Graphs

**premotor cortex**
- Δ MR Signal
- 2 Responses
- 1 Response

**posterior lateral PFC**
- Δ MR Signal
- 2 Responses
- 1 Response

**anterior lateral PFC**
- Line graph
- Δ MR Signal
Is There Anything To This Mapping?

Koechlin, Ody, & Kouneiher (2003)

**premotor cortex**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>1 Response</th>
<th>2 Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**posterior lateral PFC**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>1 Response</th>
<th>2 Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**anterior lateral PFC**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>1 Response</th>
<th>2 Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Task Set Dependence**

- **R node**
- **C node**
- **T node**