Standard Deviation as a Ruler: z-scores

Lecture 8

Reading: *GW* 5 (except 5.4)

Linear Transformations

- The same variable can be recorded in different units of measure.
- A linear transformation changes the variable *y* into a new variable, *w*, using the equation $w=a+by$.
- *a* shifts and *b* scales the new variable

Ex. Let *y* be the temperature in Fahrenheit
Let *w* the same temperature in Celsius, then

$$w = -\frac{160}{9} + \frac{5}{9}y$$

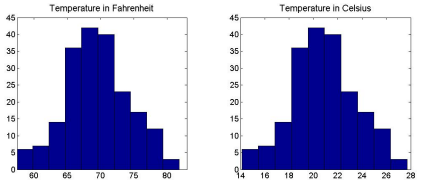
(Here a=-160/9, b=5/9).

Effects of linear transformations on distributions

- Applying a linear transformation doesn't change the shape of a distribution.
- However, it changes the center and spread.
- It multiplies a measure of spread by $|b|$.
- It changes a measure of center into $by+a$.

Example

The temperature is measured in both Celsius and Fahrenheit.



$\bar{y} = 70$ $\bar{w} = 21.11$

The Standard Deviation as a Ruler

- We are often interested in determining how unusual an observation is, or comparing values that are measured on different units.
- We can answer these types of questions by calculating how many standard deviations away from the mean an observation lays.
- This can be achieved by using a linear transformation.

Z-score when population mean and SD are known

- Suppose y is an observation on a variable with mean μ and standard deviation σ
- The standardized value, or z-score, is

$$z = \frac{y - \mu}{\sigma}$$

- The z-score measures the distance of each data value from the mean in standard deviations.

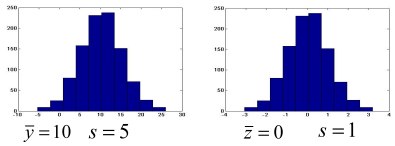
Z-score when population mean and SD are unknown

- Suppose y is an observation on a variable with mean $\bar{y}$ and standard deviation s.
- The standardized value, or z-score, is

$$z = \frac{y - \bar{y}}{s}$$

- The z-score measures the distance of each data value from the mean in standard deviations.

- Standardizing a variable doesn't change the shape of its distribution.
- However, the mean of the resulting data is 0 and the standard deviation is 1.



$\bar{y} = 10 \quad s = 5$ $\bar{z} = 0 \quad s = 1$

Question 1

- Ex. The mean on the Math portion of the SAT is 500 with s=100 and on the ACT the mean is 18 with s=6. Tom scores a 680 on the SAT and Bob scores 30 on the ACT. Who did better?

Question 2

- Ex. Jill scored a 25 on an extraversion scale. Is she an extravert?

Question 3

- Ex. Jill scored a 25 on an extraversion scale. The population mean of the scale is 40 and the population standard deviation is 10. Is she an extravert?

Question 4

- Ex. John scored a 60 on the same extraversion scale (with population mean of 40 and population standard deviation of 10). Is he extraverted?
- About what percent of people score lower than he does on extraversion?