Factorial ANOVA

Lectures 27-29

Readings: GW 15

Factorial Designs

- So far, we have been interested in the effect of a *single* independent variable [i.e., factor].
  - effects of temperature (50º, 70º, 90º) on learning
  - effects of drugs (placebo vs. aspirin) on pain
  - effects of gender (male vs. female) on pain
- What about the effects of multiple factors together?
  - Effects of drugs AND gender on pain?
  - What are the three possible effects on pain?

Two factor design example: gender and pain medication

- We induce controlled thermal pain to males and females (factor 1) who have taken a placebo or nothing (factor 2).
- Each factor has two levels: 2 x 2 design


	Males	Females
Placebo	There are many people in each cell	Here, 30 females who took placebo
Nothing	Here, there might be 30 males who took nothing	We'll put the *mean* score in each box

"Main effect" of gender


	Male	Female	
Placebo	5.4	6.4	
Nothing	7.2	6.6	
	6.3	6.5	Male vs. female

"Main effect" of placebo


	Male	Female	
Placebo	5.4	6.4	5.9
Nothing	7.2	6.6	6.9
	6.3	6.5	Placebo vs nothing

"Interaction" effect – does the effect of placebo *depend* on gender?


	Male	Female	
Placebo	5.4	6.4	5.9
Nothing	7.2	6.6	6.9
	6.3	6.5	

Difference of 1.8 — Difference of 0.2

"Interaction" effect – does the effect of placebo *depend* on gender?

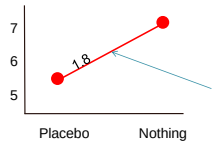
	Male	Female	
Placebo	5.4	6.4	5.9
Nothing	7.2	6.6	6.9
	6.3	6.5	

Difference of 1.8 — Difference of 0.2

The effect of placebo *depends* on gender (strong for males, not females)

Plotting two-way factorial results

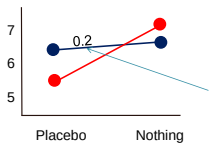
	Male ●	Female ●
Placebo	5.4	6.4
Nothing	7.2	6.6



The slope of the red line is the placebo effect among males

Plotting two-way factorial results

	Male ●	Female ●
Placebo	5.4	6.4
Nothing	7.2	6.6

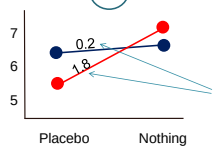


The slope of the blue line is the placebo effect among females

Plotting two-way factorial results

	Male ●	Female ●
Placebo	5.4	6.4
Nothing	7.2	6.6

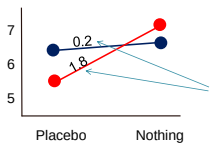
Difference of 1.8 — Difference of 0.2



Different slopes mean the effect is different for males than females – an INTERACTION!

Plotting two-way factorial results

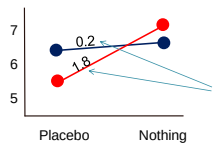
	Male ●	Female ●
Placebo	5.4	6.4
Nothing	7.2	6.6



Interaction def: the effects of one factor (placebo here) *depend* on the level of the other factor (gender here)

Plotting two-way factorial results

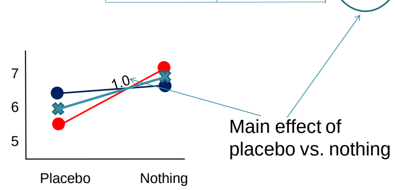
	Male ●	Female ●
Placebo	5.4	6.4
Nothing	7.2	6.6



The degree to which lines are non-parallel tell you how strong the interaction is.

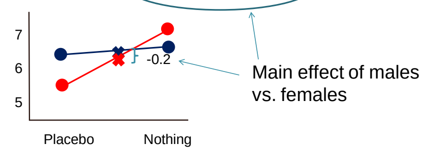
Plotting two-way factorial results

	Male●	Female●	
Placebo	5.4	6.4	5.9
Nothing	7.2	6.6	6.9



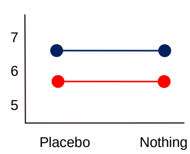
Plotting two-way factorial results

	Male●	Female●
Placebo	5.4	6.4
Nothing	7.2	6.6
	6.3	6.5



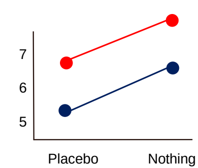
Practice at interpreting two-way factorial results

Male● Female●



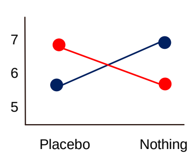
Practice at interpreting two-way factorial results

Male● Female●



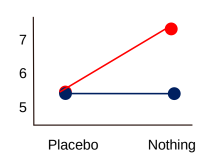
Practice at interpreting two-way factorial results

Male● Female●



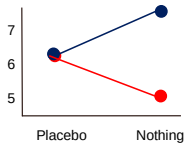
Practice at interpreting two-way factorial results

Male● Female●



Practice at interpreting two-way factorial results

Male ● Female ●



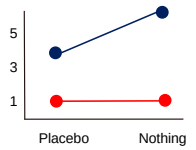
2 X 2 Factorial ANOVA

- Statistically, a 2 x 2 ANOVA is exactly like a one-way ANOVA with 4 levels, except we split the between-group Sums of Squares (SS_B) into 3 different effects

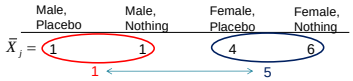
2 X 2 Factorial ANOVA

Male, Placebo	Male, Nothing	Female, Placebo	Female, Nothing
0	0	3	8
0	1	4	5
3	2	5	5
$\bar{X}_j =$ 1	1	4	6

● Male ● Female



2 X 2 Factorial ANOVA



The **3 df** for "between treatments" correspond to the 3 tests in a 2x2 ANOVA:
1. Main effect of gender

Source	SS	df	MS	
Between treatments	54	3	18	F = 9
Within treatments	16	8	2	
Total	70	11		

F_{crit} = 4.07, so reject null

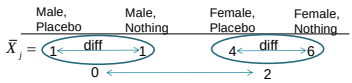
2 X 2 Factorial ANOVA

The **3 df** for "between treatments" correspond to the 3 tests in a 2x2 ANOVA:
1. Main effect of gender
2. Main effect of placebo

Source	SS	df	MS	
Between treatments	54	3	18	F = 9
Within treatments	16	8	2	
Total	70	11		

F_{crit} = 4.07, so reject null

2 X 2 Factorial ANOVA



The **3 df** for "between treatments" correspond to the 3 tests in a 2x2 ANOVA:
1. Main effect of gender
2. Main effect of placebo
3. Interaction between placebo & gender

Source	SS	df	MS	
Between treatments	54	3	18	F = 9
Within treatments	16	8	2	
Total	70	11		

F_{crit} = 4.07, so reject null

2 X 2 Factorial ANOVA

	Male, Placebo	Male, Nothing	Female, Placebo	Female, Nothing
$\bar{X}_j$ =	1	1	4	6

The **overall F** will be identical for a 2x2 ANOVA and the one-way with 4 levels:

Source	SS	df	MS	
Between treatments	54	3	18	F = 9
Within treatments	16	8	2	
Total	70	11		

F_{crit} = 4.07, so reject null

2 X 2 Factorial ANOVA

	Male, Placebo	Male, Nothing	Female, Placebo	Female, Nothing
$\bar{X}_j$ =	1	1	4	6

The only difference is that we will partition the SS into 3 effects, and get 3 Fs

Source	SS	df	MS	
Between treatments	54	3	18	F = 9
Within treatments	16	8	2	
Total	70	11		

F_{crit} = 4.07, so reject null

2 X 2 Factorial ANOVA

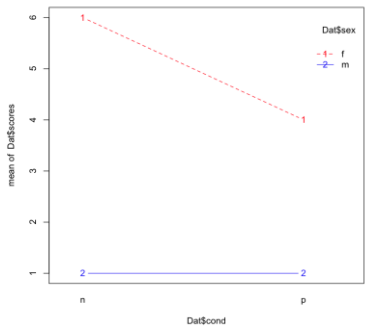
Source	SS	df	MS	
Between treatments	54	3	18	F = 9
Gender	48	1	48	F=24
Placebo	3	1	3	F=1.5
Gender x Placebo	3	1	3	F=1.5
Within treatments	16	8	2	
Total	70	11		

Factorial ANOVA in R

```
> Dat
   scores sex cond
1       0   m    p
2       0   m    p
3       3   m    p
4       0   m    n
5       1   m    n
6       2   m    n
7       3   f    p
8       4   f    p
9       5   f    p
10      8   f    n
11      5   f    n
12      5   f    n
> summary(aov(scores ~ sex * cond, data=Dat))
            Df Sum Sq Mean Sq F value Pr(>F)
sex          1     48      48    24.0 0.0012 **
cond         1      3       3     1.5 0.2555
sex:cond     1      3       3     1.5 0.2555
Residuals    8     16       2
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Factorial ANOVA in R

```
> interaction.plot(Dat$cond,Dat$sex,Dat$scores,type='b',col=c('red','blue'))
```



"4" sentence summary of Factorial ANOVA

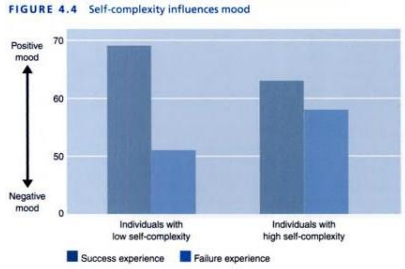
It has previously been observed that females show a stronger placebo effect than males. To test this in a pain paradigm, we selected 60 females and 60 males and randomly assigned 30 of each gender to take a placebo or nothing and recorded self-report pain (1-9) after administration of thermal pain. Overall, pain scores for females (mean of 7.2) were significantly higher than pain scores of males (mean of 6.6; F(1,116)=7.3, p=.004). Placebo also lowered pain (mean of 6.1) over taking nothing (mean of 6.9; F(1,116)=7.9,p=.003). However, the placebo effect was stronger in males than in females, as evidenced by a significant gender-by-placebo interaction (F(1,116)=6.2,p=.007). We conclude that, contrary to previous research, males show a stronger placebo effect in a thermal pain paradigm than do females.

Factorial ANOVA Example 1

- Keller et al, 2004: Researchers were interested in the effects of nice (sunny, warm) weather on mood. They found that nice weather was associated with higher mood overall, and that being outside was associated with higher mood overall. However, the effect of nice weather was stronger (more positive) among people who spent 30 minutes or more outside than it was among people who spent 30 minutes or less outside.

Factorial ANOVA Example 2

- Linville, 1985:



Factorial ANOVA Example 3

- Festinger & Carlsmith, 1959: Researchers had participants do a boring task. They then paid each participant to tell the next participant that the experiment was enjoyable. Contrary to a behaviorist model, participants who were paid $1 actually enjoyed participating in the study more than those paid $20, providing support for the cognitive dissonance theory. In other words, their level of enjoyment depended on whether they were paid $1 or $20.

Factorial ANOVA Example 4

- Harmon-Jones et al, 1996: Researchers had participants either taste a delicious or a nasty-tasting drink and write a sentence saying they liked it. Some participants were given less freedom than others about writing the sentence. Among those who tasted the pleasant drink, the amount of freedom in writing the sentence had no effect on their ratings of the drink – all participants reported liking the drink. However, among those who tasted the unpleasant drink, the amount of freedom had a big effect, such that those with no freedom reported not liking the drink whereas those with a little freedom reported liking the drink.

Factorial ANOVA – R example

```
> summary(aov(extraversion ~ girl_boyfriend * gender,data=dat2))
                     Df Sum Sq Mean Sq F value  Pr(>F)
girl_boyfriend        1   0.15   0.146  0.0241 0.87693
gender                1   6.79   6.790  1.1184 0.29277
girl_boyfriend:gender 1  33.91  33.911  5.5851 0.02001 *
Residuals           102 619.31   6.072
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```