Repeated-Measures ANOVA

Lecture 20

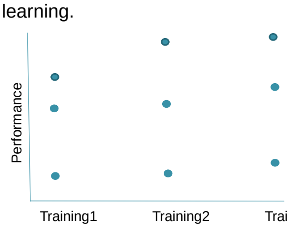
Readings: GW 14

Repeated-Measures Design

- Multiple measurements for each subject
  - Different stimulus types, conditions, times, etc.
  - All measurements are of the same variable, but in different situations
  - Generalizes paired-samples design
- Is there an effect of the treatment?
  - Variation due to condition, time, stimulus, etc.
  - Do the means of the measurements vary?
- Same null hypothesis as simple ANOVA
  - $\mu_1 = \mu_2 = \ldots = \mu_k$

Example: Repeated-Measures ANOVA

- Suppose we are interested in different training regimens on the learning of dogs
- We give several dogs the same training regimens and give them a test at weeks 2, 4, and 6 to gauge their learning.



Is there an effect of time on performance?

It's hard to tell because there is a lot of "noise" or variation in scores.

Example: Repeated-Measures ANOVA

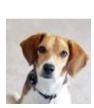
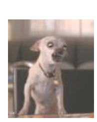
- Suppose we are interested in different training regimens on the learning of dogs
- We give several dogs the same training regimens and give them a test at weeks 2, 4, and 6 to gauge their learning.

Now say we know that the same 3 dogs were measured at each time.

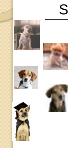
The variation between individuals is not of interest. Would be nice to "remove" it.

Repeated-Measures ANOVA

- The goal of repeated measures ANOVA is to identify and remove variation (sums of squares) due to individual differences.
- It is used when the same individual is measured 3+ times, and is more sensitive than regular ANOVA if there are stable individual differences

Repeated-Measures Data



	Measurement			
Subject	1	2	3	4
1	78	73	82	75
2	108	105	113	106
3	84	79	89	80
4	94	88	98	92
5	121	115	123	117

- Individual differences
  - Variation from one subject to another
  - Affects all the scores of any given subject

Accounting for Individual Differences

- Individual differences complicate hypothesis testing
  - Inflate variability of scores
  - Don't affect random variability of treatment means
    - Contribute to all measurements equally
- Basic idea
  - Subtract subject mean for each score
  - Do simple ANOVA on these differences (df_{error} changes)

	Measurement			
Subject	1	2	3	4
1	78	73	82	75
2	108	105	113	106
3	84	79	89	80
4	94	88	98	92
5	121	115	123	117

Accounting for Individual Differences

	Measurement					Difference Score			
Subject	1	2	3	4	M_s	1	2	3	4
1	78	73	82	75	77	1	-4	5	-2
2	108	105	113	106	108	0	-3	5	-2
3	84	79	89	80	83	1	-4	6	-3
4	94	88	98	92	93	1	-5	5	-1
5	121	115	123	117	119	2	-4	4	-2

$$SS_{total} = \sum_i \left(X_i - \bar{\bar{X}} \right)^2 = (78-96)^2 + (108-96)^2 + \ldots$$

$$SS_{total} = 5086$$

$$SS_{within} = SS_{error} = \sum_{j=1}^{k} SS_j = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_i - \bar{X}_j)^2 = (1-1)^2 + (0-1)^2 + \ldots$$

$$SS_{error} = 8$$

$$SS_{between} = SS_{treatment} = \sum_{j=1}^{k} n_j (\bar{X}_j - \bar{\bar{X}})^2 = 5(1-0)^2 + 5(-4-0)^2 + \ldots = 230$$

Repeated Measures ANOVA

	Measurement					Difference Score			
Subject	1	2	3	4	M_s	1	2	3	4
1	78	73	82	75	77	1	-4	5	-2
2	108	105	113	106	108	0	-3	5	-2
3	84	79	89	80	83	1	-4	6	-3
4	94	88	98	92	93	1	-5	5	-1
5	121	115	123	117	119	2	-4	4	-2

$$SS_{total} = 5086 \qquad df_{total} = N - 1 = nk - 1$$

$$SS_{error} = 8 \qquad df_{error} = nk - 1 - (k-1) - (n-1) = nk - n - k + 1$$

$$SS_{treatment} = 230 \qquad df_{treatment} = k - 1$$

$$SS_{between.subject} = SS_{total} - SS_{treatment} - SS_{error} = 4848 \qquad df_{between.subject} = n - 1$$

$$MS_{treatment} = SS_{treatment} / df_{treatment}$$

$$MS_{error} = SS_{error} / df_{error}$$

$$F(df_{treatment}, df_{error}) = MS_{treatment} / MS_{error}$$

Partitioning Variability

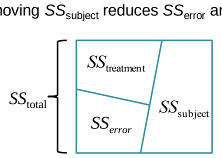
- Break total variability into treatment, subjects, and "residual" or "error" variation

$$SS_{total} = \underbrace{SS_{treatment}}_{SS_{between}} + \underbrace{SS_{subject} + SS_{error}}_{SS_{within}}$$

Because the F ratio is divided by MS_{error}, not MS_{within}, we identify and remove the variability due to subjects in Repeated Measures ANOVA

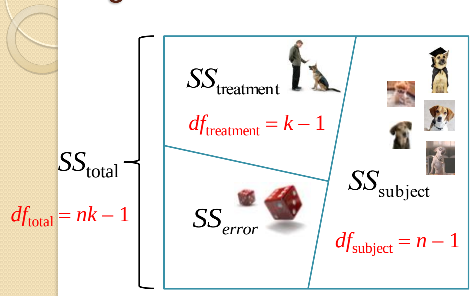
Repeated-Measures ANOVA

- Does treatment explain significant portion of variability?
  - Don't want to penalize for variability due to individual differences
  - Removing $SS_{subject}$ reduces SS_{error} and makes it a fair comparison



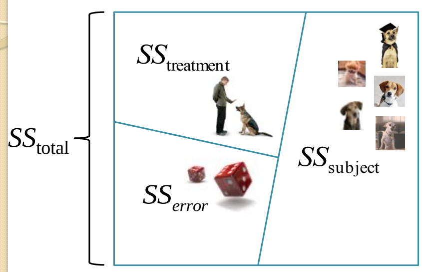
- Test for repeated-measures ANOVA: $F = \dfrac{MS_{treatment}}{MS_{error}}$
  - Same as regular ANOVA, except we first remove $SS_{subject}$
  - ($SS_{subject}$ not meaningful with simple ANOVA because each subject is only in one group)

Degrees of Freedom



$$df_{error} = nk - 1 - (k-1) - (n-1)$$
$$= nk - n - k + 1$$

Effect size



$$\eta^2 = \frac{SS_{treatment}}{SS_{treatment} + SS_{error}}$$

"The percentage of variability in scores due to treatment differences, after removing the effects of individual differences."

Results

- Regular one-way ANOVA, INCORRECTLY not accounting for subject variance:

```
> summary(aov(score~time,data=RepDat))
            Df Sum Sq Mean Sq F value Pr(>F)
time         3    230   76.67   0.253  0.858
Residuals   16   4856  303.50
```

- Repeated measures ANOVA, CORRECTLY accounting for (removing) subject variance from denominator:

```
> summary(aov(score~time + Error(person),data=RepDat))

Error: person
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals  4   4848    1212

Error: Within
          Df Sum Sq Mean Sq F value   Pr(>F)
time       3    230   76.67     115 4.17e-09 ***
Residuals 12      8    0.67
```

4-sentence summary