

## Psychophysical Scaling

### Fechner's Law (Fechner, 1860)

$R$  = unit of stimulus measurement (Reiz)

$S$  = unit of subjective measurement

$K, k, C$  and  $c$  are constants

$$(1) \quad \frac{\partial R}{R} = K \quad (\text{for and at the JND})$$

E. H. Weber's Law

$$(2) \quad \partial S = c \cdot \frac{\partial R}{R}$$

Fundamentalformel

Integrate to get the magnitude of  $S$  (sensation)

$$(3) \quad S = c \cdot \ln R + C$$

At threshold,  $S = 0$ ,  $R = r$ , therefore:  $0 = c \cdot \ln r + C$  or  $C = -c \cdot \ln r$

Substituting this relationship into (3):

$S = c \cdot \ln R - c \cdot \ln r$  or  $S = c \cdot (\ln R - \ln r)$  or

$$(4) \quad S = c \cdot \ln \left( \frac{R}{r} \right)$$

Converting Equation 4 from natural logarithms to base 10 logarithms

$$(5) \quad S = k \cdot \log \left( \frac{R}{r} \right)$$

Massformel

Making  $r$  (the threshold stimulus value) the unit of  $R$ , we have:

$$(6) \quad S = k \cdot \log R$$

Fechner's Law

Conditions for the validity of Fechner's Law:

1. Unit of  $R$  is  $r$  (threshold value)
2. Integration of Equation 2 is valid
3. Valid to assume that  $S = 0$  at  $R = r$
4. That Weber's Law is valid

Major Criticisms

1. All JNDs do not give rise to equal sensation magnitudes
2. Sensations don't only have magnitude, but also quality (e.g., color, pitch)

### Stevens' Law (Stevens, 1936)

20 JND's above auditory threshold do not sound twice as loud as 10 JND's above threshold.

Prothetic Continua

$$S = k \cdot R^n$$

$$S = k \cdot (R - r)^n$$

$$\log S = \log k + n \cdot \log R$$

Stevens' Law

Metathetic Continua

$$S = k \cdot \log R$$

Fechner's Law

### General Psychophysical Law (Norwich & Wong, 1997)

Both Fechner's Law and Stevens' Law can be viewed as special cases of a more general law based on information processing principles.

$$S = \frac{1}{2} \cdot k \cdot \ln(1 + \gamma \cdot R^n)$$

Complete Form

Note that when  $\gamma \cdot R^n \gg 1$  the complete form becomes

$$S = \frac{1}{2} \cdot k \cdot n \cdot \ln R + \frac{1}{2} \cdot k \cdot \ln \gamma$$

Equivalent to Fechner's Law

which is really Fechner's Law since  $\frac{1}{2}kn$  and  $\frac{1}{2}k \ln \gamma$  are constants.

The Taylor series expansion of the complete form has a first-order term of

$$S = \frac{1}{2} \cdot k \cdot \gamma \cdot R^n$$

Equivalent to Stevens' Law

### References

- Fechner, G. T. (1860). *Elemente der Psychophysik*. Leipzig, Germany: Breitkopf and Härtel.
- Norwich, K. H., & Wong, W. (1997). Unification of psychophysical phenomena: The complete form of Fechner's law. *Perception & Psychophysics*, 59(6), 929–940.
- Stevens, S. S. (1936). A scale for the measurement of a psychological magnitude: loudness. *Psychological Review*, 43, 405–416.